6.6 Area Between Two Curves

- Area between Two Curves
- Examples

Exam:

- Wednesday 5/8/2019; 3:30PM - 5:30PM; BUSN 106

WebAssign on this section:

- Sunday 5/5/2019

Warm Up:

- Evaluate the integral
$$\int (2+x^2)(3-\sqrt{x})\,dx$$
- Evaluate the integral
$$\int (2x+1)\sqrt[3]{x^2+x}\,dx$$

Warm Up (Cont)

Area between Two Curves

- Area Between Two Curves

> Let $y = f(x)$ and $y = g(x)$ be two continuous functions with $f(x) \geq g(x)$ on $[a, b]$. Then the area between the graphs of the two curves on $[a, b]$ is given by the definite integral
> $$\int_a^b [f(x) - g(x)]\, dx$$

- Area of a curve under the x-Axis

> If the graph of $y = f(x)$ is below the x-axis on $[a, b]$, then the area below the x-axis and above the graph of $y = f(x)$ on $[a, b]$ is
> $$\text{area} = -\int_a^b f(x)\, dx$$

Examples

(1) Find the area between $f(x) = x^2$ and $g(x) = x^3$ from $x = 0$ to $x = 1$. 1/12

1. Find intersection points:

$x^2 = x^3 \to x^3 - x^2 = x^2(1-x) = 0$

$\Leftrightarrow x = 0, 1.$

2. Intervals: $[0, 1]$ (int. pts. don't break up interval in this case)

3. Integrate over each interval

$$\int_0^1 f(x) - g(x)\,dx = \int_0^1 x^2 - x^3\,dx$$

$$= \frac{1}{3}x^3 - \frac{x^4}{4}\Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{4} - (0 - 0)$$

$$= \frac{1}{12}$$

4. Area $= \left|\frac{1}{12}\right| = \frac{1}{12}$

(2) Find the area between $f(x) = x$ and $g(x) = x^2$ from $x = 0$ to $x = 3$. Be careful with determining which curve is the bottom and which is the top.

1. Find intersection points:

$x = x^2 \to x^2 - x = x(x-1) = 0$

$\Leftrightarrow x = 0, 1.$

2. Intervals: $[0,3] \to [0,1], [1,3]$

3. Integrate over each interval

$[0,1]$: $\int_0^1 f(x) - g(x)dx = \int_0^1 x - x^2 dx$

$= \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_0^1$

$= \frac{1}{2} - \frac{1}{3} - (0 - 0)$

$= \frac{1}{6}$

$[1,3]$: $\int_1^3 x - x^2 dx = \frac{1}{2}x^2 - \frac{1}{3}x^3 \Big|_1^3$

$= \frac{3^2}{2} - \frac{3^3}{3} - \left(\frac{1^2}{2} - \frac{1^3}{3}\right)$

$= -\frac{14}{3}$

4. Total area $= \left|\frac{1}{6}\right| + \left|-\frac{14}{3}\right| = \frac{29}{6}$

(3) Find the area between $f(x) = x + 2$ and $g(x) = \sqrt[3]{x}$ from $x = -1$ to $x = 1$. 4

Determine the area that is bounded by the following curve and the x-axis on the interval below. (Round your answer to three decimal places.)

- $y = x^2 - 9, \qquad -6 \leq x \leq 1$ 62.667

$[-6, 1]$

↳ 2nd curve is $y = 0$

1. Find intersection points:

$x^2 - 9 = (x-3)(x+3)$

↳ $x = 3, -3$

2. Intervals: $[-6, 1] \to [-6, -3], [-3, 1]$ (3 outside of $[-6, 1]$)

3. Integrate over each interval

$[-6, -3]$: $\int_{-6}^{-3} x^2 - 9 - 0 \, dx = \int_{-6}^{-3} x^2 - 9 \, dx$

$= \frac{x^3}{3} - 9x \Big|_{-6}^{1}$

$= \frac{(-3)^3}{3} - 9(-3) - \left(\frac{(-6)^3}{3} - 9 \cdot (-6)\right)$

$= 36$

$[-3, 1]$: $\int_{-3}^{1} x^2 - 9 \, dx = \frac{x^3}{3} - 9x \Big|_{-3}^{1}$

$= \frac{1^3}{3} - 9 \cdot 1 - \left(\frac{(-3)^3}{3} - 9 \cdot (-3)\right)$

$= -\frac{80}{3}$

4. Total area $= 36 + \left|\frac{-80}{3}\right| = \frac{188}{3}$

- $y = e^{2x}, \qquad -2 \leq x \leq 1$ 3.685

Determine the area that is bounded by the graphs of the following equations. (Round your answer to three decimal places.)

- $y = 64x, \qquad y = x^3\ 2048$

1. Find intersection points:

$64x = x^3 \rightarrow \quad x^3 - 64x = x(x^2-64) = x(x-8)(x+8) = 0$

$\hookrightarrow \; x = 0, 8, -8$

2. Intervals: $[-8, 0], [0, 8]$

3. Integrate over each interval

$[-8,0]$: $\displaystyle\int_{-8}^{0} 64x - x^3 dx = 32x^2 - \frac{1}{4}x^4 \Big|_{-8}^{0}$

$= (32 \cdot 0 - \frac{1}{4} \cdot 0) - (32 \cdot 8^2 - \frac{1}{4} 8^4)$

$= -1024$

$[0,8]$: $\displaystyle\int_{0}^{8} 64x - x^3 dx = 32x^2 - \frac{1}{4}x^4 \Big|_{0}^{8}$

$= 32 \cdot 8^2 - \frac{1}{4} \cdot 8^4 - (32 \cdot 0^2 - \frac{1}{4} 0^4)$

$= 1024$

4. Total area $= |-1024| + 1024 = 2048$

- $y = 3x, \qquad y = 5x - x^2$ 1.333

1. Find intersection points:

$3x = 5x - x^2 \rightarrow 0 = 2x - x^2 = (2-x)x$

$\hookrightarrow x = 0, 2$

2. Intervals: $[0, 2]$

3. Integrate over each interval:

$[0, 2]$: $\int_0^2 3x - (5x - x^2)\,dx = \int_0^2 -2x + x^2\,dx$

$= -x^2 + \frac{1}{3}x^3 \Big|_0^2$

$= -2^2 + \frac{1}{3}2^3 - (-0^2 + \frac{1}{3}0^3)$

$= -4 + \frac{8}{3}$

$= -\frac{4}{3}$

4. Total area $= |-\frac{4}{3}| = \frac{4}{3}$

- $y = -x^2, \qquad y = x^3 - 6x$ 21.083

Determine the area that is bounded by the graphs of the following equations on the interval below. (Round your answer to three decimal places.)

- $y = x^2 + 7x, \qquad y = 8x + 56, \qquad -4 \leq x \leq 6$ 476.667

- $y = 14/x, \qquad y = 7, \qquad 1 \le x \le 7$ 20.165

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The graph of f is shown. Evaluate each integral by interpreting it in terms of areas.

- $\int_{20}^{28} f(x)\,dx$
- $\int_{0}^{36} f(x)\,dx$