Math 1071Q Integration Worksheet

Name:

Power Rule

The power rule for indefinite integrals is

$$\int x^p dx = \frac{1}{p+1}x^{p+1} + C, \quad p \neq -1.$$

Problem 1. For the following integrals, fill in the blank for the power rule.

1. $$\int x^7 dx = \frac{1}{\_+1}x^{\_+1} + \_$$
2. $$\int \frac{2}{x^2} dx = \frac{\_}{\_+1}x^{\_+1} + \_$$
3. $$\int x^{\_} dx = \frac{1}{8}x^8 + \_$$

Other Rules

The other integration rules are

$$\int x^{-1} dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int f(x) + g(x) dx = \int f(x) dx + \int g(x) dx$$

$$\int af(x) dx = a \int f(x) dx$$

Problem 2. Use these rules to compute the following integrals.

1. $$\int x^{-2} - \frac{3}{x} dx =$$
2. $$\int -e^x - x^{-0.5} dx =$$
3. $$\int x(x^5 - 3) dx =$$

U-Substitution

An example of u-substitution is given:

To compute

$$\int xe^{2x^2+1}dx,$$

set $u = 2x^2 + 1$. Differentiating with respect to x gives $\frac{du}{dx} = 4x$. Solving for dx gives: $dx = \frac{1}{4x}du$. Thus, by substituting $u = 2x^2 + 1$ and $dx = \frac{1}{4x}du$, we get

$$\int xe^{2x^2+1}dx = \int xe^u dx = \int xe^u \frac{1}{4x}du = \int \frac{1}{4}e^u du = \frac{1}{4}e^u + C = \frac{1}{4}e^{2x^2+1} + C.$$

Problem 3. For these problems, fill in the blanks and compute the integrals.

1.

$$\int x^2 e^{x^3-2}dx \implies \begin{array}{c} u = \\ \frac{du}{dx} = \\ dx = \end{array}$$

$$\implies \int x^2 e^{x^3-2}dx =$$

2.

$$\int (3x^2 - 2x)(x^3 - x^2)^8 dx \implies \begin{array}{c} u = \\ \frac{du}{dx} = \\ dx = \end{array}$$

$$\implies \int (3x^2 - 2x)(x^3 - x^2)^8 dx =$$

3.

$$\int \frac{3x^2}{x^3 - 1}dx \implies \begin{array}{c} u = \\ \frac{du}{dx} = \\ dx = \end{array}$$

$$\implies \int \frac{3x^2}{x^3 - 1}dx =$$