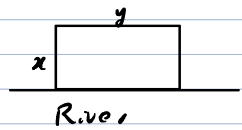
Section 5.6: Optimization and Modeling

1.



1. $500 = 2x + y$

   $A = x \cdot y$

2. $y = 500 - 2x$, $A(x) = x(500 - 2x) = 500x - 2x^2$

3. $y = 0 \to 2x = 500 \to x = 250$

   $y = 500 \to 2x + 500 = 500 \to x = 0$

   So $0 < x < 250$

4. Find abs. ext.: $A' = 500 - 4x = 0$ when $x = \frac{500}{4} = 125$

	0	125	250
A	0	31250	0

$A(0) = 0$

$A(125) = 125(500 - 2 \cdot 125) = 31250$

5. A max. when $x = 125$, $y = 500 - 2 \cdot 125 = 250$.

2)

1. $p = 5 \to x = 250$

   $x = mp + b \to m = \frac{10 \text{ tickets}}{1 \text{ dollar}} = -10$

   $x = -10p + b$

   At $(5, 250)$: $250 = -10 \cdot 5 + b = -50 + b \to b = 300$

   $x = -10p + 300$

2. $R(p) = px = p(-10p + 300) = -10p^2 + 300p$.

3. smallest $p = 0$.

   largest p is when $x = 0$: $0 = -10p + 300 \to p = 30$.

   Thus $0 < p < 30$.

4. $R' = -20p + 300 = 0$ when $p = 15$.

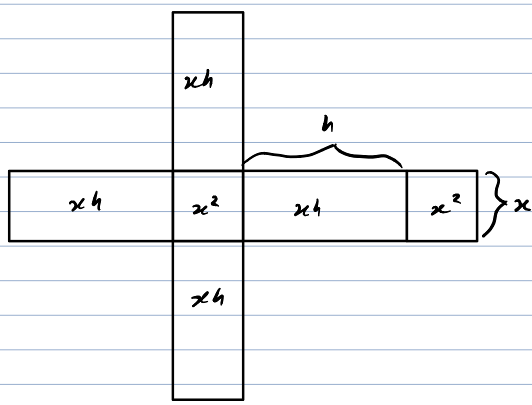
	(	$\vert$	)
	0	15	30
R	0	2250	0

$R(0) = R(30) = 0$
$R(15) = 2250$

Max $R = 2250$

5. Must charge $p = 15$ to max. revenue.

3)
1.



$V = 128 = x \cdot x \cdot h = x^2 h$

$C = 2$(Area of 4 sides and top)
$+ 6$(Area of bottom)
$= 2(4 \cdot xh + x^2) + 6x^2$
$= 8xh + 2x^2 + 6x^2$
$= 8xh + 8x^2.$

2. $128 = x^2 h \rightarrow \quad h = \frac{128}{x^2} \rightarrow \quad C = 8 \cdot x \frac{128}{x^2} + 8x^2$
$= \frac{1024}{x} + 8x^2$

3. Note $0 < x < +\infty.$

4. $C' = -\frac{1024}{x^2} + 16x = 0$

$16x = \frac{1024}{x^2}$

$x^3 = \frac{1024}{16} = 64$

$x = 4$

$C(x) = \frac{1024}{x} + 8x^2$

	(	$\vert$	)
	0	4	$+\infty$
C	$+\infty$	384	$+\infty$

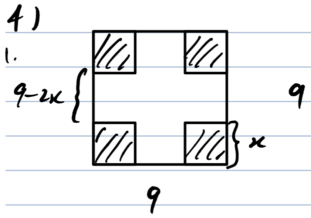
$C(0) = +\infty$
$C(4) = 384$
$C(+\infty) = +\infty$

Thus min Cost $= 384$

5. Dim. to min. : $x = 4$, $h = \frac{128}{(4)^2} = 8$

4)

1.



2. $V = (9-2x)^2 \cdot x = 4x^3 - 36x^2 + 81x$

3. $0 < x < 4.5$

cut nothing   cut a full corner.

4. $V' = 12x^2 - 72x + 81 = 0 \rightarrow x = 1.5,\ 4.5$

	(	1.5	)
	0	1.5	4.5
V	0	54	0

$V(0) = 0$
$V(4.5) = 0$
$V(1.5) = 54$

5. Vol. is maximized when $x = 54$.