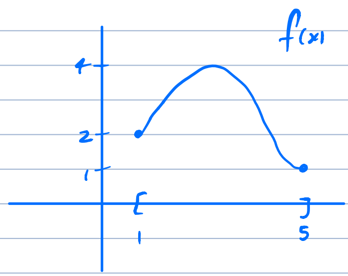
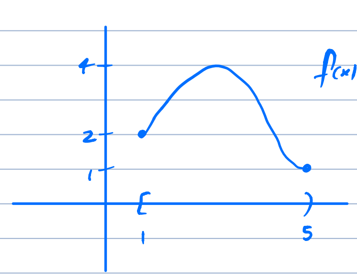
Section 5.5 Absolute Extrema

Absolute max = max value of function
Absolute min = min value of function

Warning: Abs. ext. will be with respect to a specified interval

Example





Abs. ext. on $[1,5]$:
Abs. max. $= 4$
Abs. min $= 1$

Abs. ext. on $[1,5)$
Abs. max. $= 4$
Abs. min. DNE
↳ since $x=5$ is not allowed
$f(5)=1$ is never attained on
$[1,5)$

Fact: If f is continuous on $[a,b]$, the $f(x)$ has a abs. max. and abs. min on $[a,b]$.
The same cannot be said for non-closed intervals, or discontinuous functions.

Problem
1) $f(x) = x^2 - 2x + 1$
Find abs. ext. on
a) $(0, 2)$

1. $f'(x) = 2x - 2 = 2(x-1) = 0$ when $x = 1$

2.

	0 (	1	) 2
$f(x)$	1	0	1

$f(0) = 0^2 - 2\cdot 0 + 1 = 1$
$f(1) = 1 - 2 + 1 = 0$
$f(2) = 4 - 4 + 1 = 1$

3. $f(1) = 0$ is abs. min.
No abs. max. since $x = 0, 2$ do not belong to $(0,2)$

b) $(0,2]$

1. as above

2. $\overset{0}{(}\!\!-\!\!-\!\!\overset{1}{\bullet}\!\!-\!\!-\!\!\overset{2}{]}$

$f(x)$ 1   0   1

3. $f(1)=0$ is abs. min
$f(2)=1$ is abs. max.

2) $f(x) = \frac{1}{3}x^3 - x$
Find abs. ext. on
a) $[0,5]$

1. $f'(x) = x^2 - 1 = 0$ when $x = 1, -1$

2. $\overset{0}{[}\!\!-\!\!-\!\!\overset{1}{\bullet}\!\!-\!\!-\!\!\overset{5}{]}$

$f(x)$ 0 $-\frac{2}{3}$ $110/3$

$f(0) = 0$
$f(1) = \frac{1}{3} - 1 = -\frac{2}{3}$
$f(5) = 110/3$

Note $x=-1$ is not in $[0,5] \longrightarrow$ ignore it.

3. $f(1) = -\frac{2}{3}$ = abs. min
$f(5) = 110/3$ = abs. max.

3) $f(x) = x - \ln(4x)$
Find abs. ext. on $[\frac{1}{2}, 2]$:

1. $f'(x) = 1 - \frac{1}{x} = 0 \rightarrow x - 1 = 0 \rightarrow x = 1$

2. $\overset{\frac{1}{2}}{[}\!\!-\!\!-\!\!\overset{1}{\bullet}\!\!-\!\!-\!\!\overset{2}{]}$

$f(x)$

$f(\frac{1}{2}) = \frac{1}{2} - \ln 2 \approx -0.19$
$f(1) = 1 - \ln 4 \approx -0.39$
$f(2) = 2 - \ln 8 \approx -0.079$

3. $f(1) = 1 - \ln 4$ = abs. min.
$f(2) = 2 - \ln 8$ = abs. max.

4) $f(x) = \frac{x^8}{e^x}$
Find abs. max on $[1, \infty)$

1. $f'(x) = \frac{e^x \cdot 8x^7 - x^8 e^x}{e^{2x}} = \frac{x^7 e^x (8 - x)}{e^{2x}} = \frac{x^7(8-x)}{e^x} = 0$

$\hookrightarrow$ $x = 0$ or $x = 8$

2. $[1 \quad 8 \quad +\infty)$

$f(x)$

$f(1) = \frac{1}{e}$
$f(8) \approx 5628$
$f(+\infty) = \lim_{x\to+\infty} \frac{x^8}{e^x} = 0$

Hint : e^x dominates all polynomials

3. No abs. min f tends to 0 as $x \to +\infty$

$f(8)$ = abs. max.

5) $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$

Find abs. ext. on $[0, 3)$

1. $f'(x) = x^2 - 3x + 2 = (x-1)(x-2) = 0 \to x = 1, 2$

2. $[0 \quad 1 \quad 2 \quad 3)$

$f(x) \quad 0 \quad \frac{5}{6} \quad \frac{2}{3} \quad \frac{3}{2}$

$f(0) = 0$
$f(1) = \frac{5}{6}$
$f(2) = \frac{2}{3}$
$f(3) = \frac{3}{2}$

3. $f(0) = 0$ = abs. min
No abs. max since $f(3)$ is largest but 3 not in $[0, 3)$.

$$c \int (u+t\varphi)^2 F_1 \le \int |\nabla(u+t\varphi)|^2 F_2 = \int \cancel{|\nabla u|^2} F_2 + 2\int t(\nabla u \cdot \nabla\varphi) F_2 + \int t^2 \cancel{|\nabla\varphi|^2} F_2$$

$$\cancel{\int u^2 F_1} + 2\int t u\varphi F_1 + \int \cancel{t^2\varphi^2} F_1$$

$$2\int u\varphi F_1 \text{ - - - - } \quad 2\int \nabla u \cdot \nabla\varphi F_2$$

$v(t) = u(\varepsilon t)$
$v_t(t) = \varepsilon u_t(\varepsilon t)$

$\int v_t^2(t) = \varepsilon^2 \int u_t(\varepsilon t)$

$$\frac{\int v_t^2 + \alpha v^2}{\int v^2} = \frac{\int v_t^2}{\int v^2} + \alpha = \varepsilon^2 \frac{\int u_t^2}{\int u^2}$$