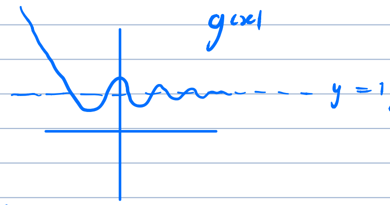
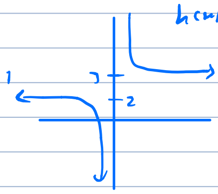
# Section 5.3 Limits at Infinity

**Examples**


$\lim_{x \to +\infty} f(x) = +\infty$ $\quad$ $\lim_{x \to +\infty} g(x) = 1$ $\quad$ $\lim_{x \to +\infty} h(x) = 3$

$\lim_{x \to -\infty} f(x) = 0$ $\quad$ $\lim_{x \to -\infty} g(x) = +\infty$ $\quad$ $\lim_{x \to -\infty} h(x) = 2$

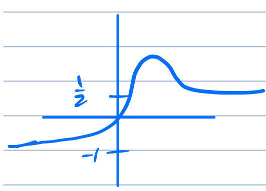
## Horizontal asymptotes

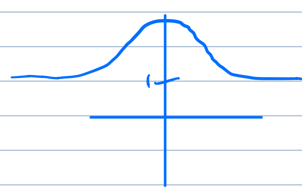
If $\lim_{x \to +\infty} f(x) = c$ and $c \neq \pm\infty$, then $y = c$ = Horizontal asymptote (H.A.)

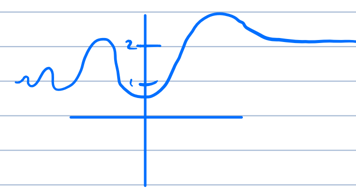
Similar for $\lim_{x \to -\infty} f(x) = c$

Warning: graphs may intersect H.A.s

**Examples**




HAs: $y = -1$, $y = \frac{1}{2}$ $\quad$ HA: $y = 1$ $\quad$ HAs: $y = 1$, $y = 2$.

## Computing limits at $\pm\infty$.

**Example**

$\lim_{x \to +\infty} x^2 + x^3$ = "Large pos #" + "larger pos #" $= +\infty$

$\lim_{x \to -\infty} x^2 + x^3$ = "Large pos #" + "larger neg #" $= -\infty$

$\lim_{x \to +\infty} x^2 - x^3$ = "Large pos #" − "larger pos #" $= -\infty$

$\lim_{x \to -\infty} x^2 - x^3$ = "Large pos #" − "larger neg. #" $= +\infty$

Example

$$\lim_{x\to\pm\infty} \frac{1}{x^p} \text{ "=" } \frac{1}{\text{large } \pm\#} = 0 \text{ for any } p>0$$

Problem

1) Compute

$$\lim_{x\to+\infty} \frac{11x^5 - x + 5}{4x^5 + 5x^4 - 8}$$

Warning DO NOT plug in $+\infty$

Tip: Divide by highest power in denominator:

$$\lim_{x\to+\infty} \frac{11x^5 - x + 5}{4x^5 + 5x^4 - 8} = \lim_{x\to+\infty} \frac{(11x^5 - x + 5)/x^5}{(4x^5 + 5x^4 - 8)/x^5} = \lim_{x\to+\infty} \frac{11 - 1/x^4 + 5/x^5}{4 + 5/x - 8/x^5}$$

$$= \frac{11 - 0 + 0}{4 + 0 - 0}$$

$$= 11/4$$

2) Compute $\displaystyle\lim_{x\to-\infty} \frac{6x^2 - 1}{5x^3 + 3}$

$$\lim_{x\to-\infty} \frac{6x^2-1}{5x^3+3} = \lim_{x\to-\infty} \frac{(6x^2-1)/x^3}{(5x^3+3)/x^3} = \lim_{x\to-\infty} \frac{6/x - 1/x^3}{5 + 3/x^3}$$

$$= \frac{0-0}{5+0}$$

$$= 0.$$

Problem

1) Find HA's of $y = \dfrac{3x^2+9}{8x^3 - 71x - 9}$

Tip: Compute $\displaystyle\lim_{x\to\pm\infty} y = c$. If $c \neq \pm\infty$, the $y=c$ a HA.

$$\lim_{x\to+\infty} \frac{3x^2+9}{8x^3-71x-9} = \lim_{x\to+\infty} \frac{(3x^2+9)/x^3}{(8x^3-71x-9)/x^3} = \lim_{x\to+\infty} \frac{3/x + 9/x^3}{8 - 71/x^2 - 9/x^3}$$

$$= \frac{0+0}{8-0-0}$$

$$= 0.$$

Thus $y=0$ is an HA.

Similarly: $\displaystyle\lim_{x\to-\infty} \frac{3x^2+9}{8x^3-71x-9} = 0.$

2) Compute

$$\lim_{x \to -\infty} \frac{x^5 + x^2}{x^2 + 1}$$

Divide by $x^2$:

$$\lim_{x \to -\infty} \frac{x^5 + x^2}{x^2+1} = \lim_{x \to -\infty} \frac{x^5/x^2 + x^2/x^2}{x^2/x^2 + 1/x^2} = \lim_{x \to -\infty} \frac{x^3 + 1}{1 + 1/x^2} = \frac{\text{large } - \#}{1+0} = -\infty.$$