

Section 4.5 Elasticity of Demand

Demand $x = f(p)$

$$\text{Elasticity of Demand } E = \frac{\% \text{ change in demand}}{\% \text{ change in price}}$$
$$= - \frac{dx/x}{dp/p}$$

$$= - \frac{p}{x} \frac{dx}{dp} \quad \begin{array}{l} \text{— always a function of } p. \\ \text{— always positive.} \end{array}$$

$E > 1 \iff$ demand is elastic

$E < 1$ " " inelastic

$E = 1$ " " of unit elasticity

$E > 1 : p \uparrow \Rightarrow R \downarrow$

$E < 1 : p \uparrow \Rightarrow R \downarrow$

$E = 1 : R$ is maximized

Example

1) Demand eq. $p = -x + 7$.

a) Find elasticity of demand:

Want $E = - \frac{p}{x} \frac{dx}{dp}$

$$p = -x + 7$$

$$x = -p + 7$$

$$\frac{dx}{dp} = -1$$

$$\frac{dp}{dp}$$

$$E(p) = - \frac{p}{-p+7} \cdot -1 = \frac{p}{-p+7}$$

b) Find E when $p=6$ and interpret:

$$E(6) = \frac{6}{-6+7} = 6 > 1$$

• $E(6) > 1 : p \uparrow \Rightarrow R \downarrow$

• $E(6) = \frac{\% \text{ change in demand}}{\% \text{ change in price}} = 6$

$\Rightarrow$ demand is going down by 6% per 1% increase in price

always opposite.

c) What price maximizes Revenue?

Recall: Revenue is maximized when $E(p) = 1$.

$\hookrightarrow$

need to solve $E(p) = \frac{p}{-p+7} = 1 \rightarrow p = -p+7 \rightarrow p = \frac{7}{2}$.

2) Assume $E(83) = 0.61$. If price is decreased by \$6, then what is approximate change in demand?

$$\text{Original price: } p = 83.6$$
$$\% \text{ decrease in } p: \frac{6}{83} \approx 7.2 \%$$

$$0.61 = E(83) = \frac{\% \text{ change in demand}}{\% \text{ change in price}} = \frac{\% \text{ change in demand}}{7.2}$$

$$\hookrightarrow \% \text{ change in demand} \approx 4.4 \%$$

Warning: We assign q in place of x for demand.

Problems

1) $x(p) = (118 - 3p)^2$.

a) Find p to maximize Revenue

b) Compute $E(3)$ and interpret:

Demand is _____

As price _____, Revenue _____

demand is going _____ by _____ per 1% increase in price

2) Assume $E(20) = 3$. If demand is increased by 2%, then what is approx. % change in price?