

Section 4.1 Derivative Rules

Notation: $f'(x) = \frac{df}{dx}$

Power Rule:

$p = \text{any number}$

$$\frac{dx^p}{dx} = p x^{p-1}$$

Examples

$$\frac{d}{dx} x^3: \quad p=3 \Rightarrow \frac{d}{dx} x^3 = 3 x^{3-1} = 3x^2$$

$$\frac{d}{dx} \frac{1}{x}: \quad p=-1 \Rightarrow \frac{d}{dx} \frac{1}{x} = -1 x^{-1-1} = -x^{-2}$$

$$\frac{d}{dx} x^{-\pi}: \quad p=-\pi \Rightarrow \frac{d}{dx} x^{-\pi} = -\pi x^{-\pi-1} = -\pi x^{-\pi-1}$$

$$\frac{d}{dx} x: \quad p=1 \Rightarrow \frac{d}{dx} x = 1 x^{1-1} = x^0 = 1.$$

Constants:

$$\frac{d}{dx} c = 0 \text{ for any number } c.$$

Examples

$$\frac{d}{dx} 5 = 0 \quad \frac{d}{dx} 1 = 0 \quad \frac{d}{dx} 7^2 = 0 \quad \frac{d}{dx} \pi = 0$$

Exponentials:

$$\frac{d}{dx} e^x = e^x$$

Examples

$$\frac{d}{dx} e^x = e^x \quad \frac{d}{dx} e^x = e^x \quad \frac{d}{dx} e^x = e^x \quad \frac{d}{dx} e^x = e^x \dots$$

Logarithms:

$$\frac{d}{dx} \ln x = \frac{1}{x}$$

Examples

$$\frac{d}{dx} \ln x = \frac{1}{x} \quad \frac{d}{dx} \ln x = \frac{1}{x} \quad \frac{d}{dx} \ln x = \frac{1}{x} \dots$$

Extra Rules for $\frac{d}{dx}$:

$$\frac{d}{dx} (f(x) + g(x)) = \frac{df}{dx} + \frac{dg}{dx}$$

$$\frac{d}{dx} (a f(x)) = a \frac{df}{dx}.$$

Examples

$$1) \frac{d}{dx}(x^2 + 2x + 1) = \frac{d}{dx}x^2 + 2\frac{d}{dx}x + \frac{d}{dx}1 = 2x + 2$$

$$2) \frac{d}{dx}(3x^3 + 7\ln x) = 3\frac{d}{dx}x^3 + 7\frac{d}{dx}\ln x = 3 \cdot 3x^2 + 7 \cdot \frac{1}{x} = 9x^2 + \frac{7}{x}$$

$$3) \frac{d}{dx} \frac{x^2 + x}{x} = \frac{d}{dx}(x + 1) = 1 + 0 = 1$$

$$4) \frac{d}{dx}(e^x + x^{-3}) = e^x - 3x^{-4}$$

Tangent lines:

$$y = f'(a)(x-a) + f(a)$$

Problems

1) Find tangent line of $f(x) = e^x + x$ at $x = 0$.

$$a = 0$$

$$f(a) = f(0) = e^0 + 0 = 1$$

$$f'(x) = \frac{d}{dx}(e^x + x) = e^x + 1$$

$$f'(0) = e^0 + 1 = 1 + 1 = 2$$

$$y = f'(0)(x-0) + f(0) = 2x + 1$$

2) Find tangent line of $f(x) = \ln x - x^{-2}$ at $x = 1$.

$$a = 1$$

$$f(a) = f(1) = \ln 1 - 1^{-2} = 0 - 1 = -1$$

$$f'(x) = \frac{d}{dx}(\ln x - x^{-2}) = \frac{1}{x} + 2x^{-3}$$

$$f'(1) = \frac{1}{1} + 2 \cdot 1^{-3} = 3$$

$$y = f'(1)(x-1) + f(1) = 3(x-1) - 1 = 3x - 4$$

3) Where is tangent line of $f(x) = \ln x - x$ horizontal?

$$y = f'(a)(x-a) + f(a) \text{ horizontal} \iff \text{slope} = 0 \iff f'(a) = 0$$

$$f'(x) = \frac{d}{dx}(\ln x - x) = \frac{1}{x} - 1 = 0 \iff \frac{1}{x} = 1 \iff x = 1$$