Section A.2

Objectives:

- Manipulate and factor polynomials
- Manipulate and simplify rational expressions.

Reminder:

linear: $ax+b$

quadratic: ax^2+bx+c

cubic: ax^3+bx^2+cx+d

Polynomials

Problems

1) Expand $(2x+3)(x+1)$:

$$(2x+3)(x+1) = 2x(x+1)+3(x+1) = 2x^2+2x+3x+3 = 2x^2+5x+3$$

2) Factor $3x^2+x$:

Every term has an x $\to$ factor out an x

$\to$ $3x^2+x = x(3x+1)$

3) Solve $3x^2+x=0$

By 2)

$$3x^2+x = x(3x+1) = 0$$

$\hookrightarrow$ $x=0$ or $3x+1=0$

$\hookrightarrow$ $x=0$ or $x=-\frac{1}{3}$

4) Factor x^2-2x+1

quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Match x^2-2x+1 $\to$ $a=1, b=-2, c=1$

ax^2+bx+c

$\to$ $x = \dfrac{-(-2) \pm \sqrt{(-2)^2-4\cdot 1\cdot 1}}{2\cdot 1}$

$= \dfrac{2 \pm \sqrt{4-4}}{2\cdot 1}$

$= 1$

Therefore, root is $x=1$

$\hookrightarrow$ $x^2-2x+1 = (x-1)^2$

5) Factor $2x^2-3x+1$

quadratic formula: $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

Match $2x^2-3x+1$ $\rightarrow$ $a=2,\ b=-3,\ c=1$

ax^2+bx+c

$\rightarrow\ x=\dfrac{-(-3)\pm\sqrt{(-3)^2-4(2)(1)}}{2\cdot 2}$

$=\dfrac{3\pm\sqrt{9-8}}{}$

$=\dfrac{3\pm 1}{4}$

$=\frac{3}{4}\pm\frac{1}{4}=1$ or $\frac{1}{2}$

Therefore, roots are $x=1$ or $\frac{1}{2}$

But $a=2\rightarrow$ write $2x^2-3x+1=2(x-1)(x-\frac{1}{2})=(2x-2)(x-\frac{1}{2})$ or $(x-1)(2x-1)$

Difference of Squares

$a^2-b^2=(a-b)(a+b)$

Example: "Simplify" x^2-2:

Compare x^2-2

$\rightarrow\ x^2=a^2,\ b^2=2\ \rightarrow\ x=a,\ b=\sqrt{2}$

a^2-b^2

Therefore, by diff. of squares formula:

$x^2-2=(x-\sqrt{2})(x+\sqrt{2})$

Problem

1) Solve $x^4-16=0$

Compare x^4-16 $\rightarrow$ $a^2=x^4$, $b^2=16$ $\rightarrow$ $a=x^2$, $b=4$ (can use $a=-x^2$, $b=-4$)

a^2-b^2

Therefore

$x^4-16=0\ \rightarrow\ x^4-16=(x^2-4)(x^2+4)=0$

To solve, set each term to 0:

$x^2-4=0\rightarrow x^2=4\rightarrow x=\pm 2$

$x^2+4=0\rightarrow x^2=-4\rightarrow$ the x^2+4 term is never zero.

Therefore the solutions are $x=\pm 2$.

Rational Expressions

Recall:

- $$\frac{1}{a}+\frac{1}{b} = \frac{1}{a}\frac{b}{b} + \frac{1}{b}\frac{a}{a}$$
$$= \frac{b}{ab} + \frac{a}{ab}$$
$$= \frac{b+a}{ab}$$

- $$\frac{a}{\frac{b}{c}} = \frac{ac}{b}, \quad \frac{\frac{a}{b}}{c} = \frac{a}{bc}$$

Problems

1) Rewrite expression as single rational expression and simplify:

a) $$\frac{x^2-1}{x-7} - \frac{1}{x-1} = \frac{x^2-1}{x-7}\frac{x-1}{x-1} - \frac{1}{x-1}\frac{x-7}{x-7}$$
$$= \frac{(x^2-1)(x-1)}{(x-7)(x-1)} - \frac{(x-7)}{(x-7)(x-1)}$$
$$= \frac{(x^2-1)(x-1)-(x-7)}{(x-7)(x-1)}$$
$$= \frac{x^3-x-x^2+1-x+7}{x^2-x-7x+7}$$

Be careful with distributing negatives

$$= \frac{x^3-x^2-2x+8}{x^2-8x+7}$$

b) If $f(x)=x^2+1$, $g(x)=x+1$, $h(x)=x-1$,

$$\frac{f(x)}{\left(\frac{g(x)}{h(x)}\right)} \quad \text{and} \quad \frac{\left(\frac{f(x)}{g(x)}\right)}{h(x)}$$

$$\frac{f(x)}{\frac{g(x)}{h(x)}} = \frac{x^2+1}{\frac{x+1}{x-1}} = \frac{(x^2+1)(x-1)}{x+1} = \frac{x^3-x^2+x-1}{x-1}$$

$$\frac{\frac{f(x)}{g(x)}}{h(x)} = \frac{\frac{x^2+1}{x+1}}{x-1} = \frac{x^2+1}{(x+1)(x-1)} = \frac{x^2+1}{x^2-1}$$