Homogeneous Equations:

Homogeneous: $M(x,y)dx + N(x,y)dy = 0 \to M(tx,ty) = t^\alpha M(x,y),\ N(tx,ty) = t^\alpha N(x,y)$
Substitution: $y = ux$ or $x = vy$

1) Solve
$$(y^2 + yx)dx - x^2 dy = 0$$

Test for homogeneity: $M(x,y) = y^2 + xy \to M(tx,ty) = t^2y + t^2yx = t^2(y+xy) = t^2 M(x,y)$
$N(x,y) = -x^2 \to N(tx,ty) = -t^2x^2 = t^2(-x^2) = t^2 N(x,y)$.

Substitution: $y = ux$ (or $x = vy$), $dy = u\,dx + x\,du$

$$\begin{aligned}
(y^2 + yx)dx - x^2dy = 0 &\longleftrightarrow (u^2x^2 + ux^2)dx - x^2(u\,dx + x\,du) = 0\\
&\longleftrightarrow (u^2+u)dx - u\,dx - x\,du = 0\\
&\longleftrightarrow u^2 dx = x\,du\\
&\longleftrightarrow \tfrac{1}{x}dx = \tfrac{1}{u^2}du\\
&\longleftrightarrow \ln x = -\tfrac{1}{u} + C\\
&\longleftrightarrow \ln x = -\tfrac{x}{y} + C \quad (u = \tfrac{y}{x})
\end{aligned}$$

Note: If I ask for a solution, $\ln x = -\frac{x}{y} + C$ is fine. However if I ask for an explicit solution, you must solve for y:

$$\ln x = -\frac{x}{y} + C \to \frac{x}{y} = -\ln x + C$$
$$\to y = \frac{x}{C - \ln x}.$$

Linear Equations

Standard form: $\frac{dy}{dx} + P(x)y = f(x)$
Integrating factor: $\mu = e^{\int P(x)dx}$
Solution: $y = \frac{1}{\mu}\int \mu f\,dx + \frac{C}{\mu}$

2) Solve $2y' = y + e^{x/2}$

Standard form: $y' - \frac{1}{2}y = \frac{1}{2}e^{x/2}$, $P(x) = -\frac{1}{2}$, $f(x) = \frac{1}{2}e^{x/2}$
Integrating factor:
$$\mu = e^{\int P(x)dx} = e^{\int -\frac{1}{2}dx} = e^{-\frac{x}{2}}$$
Solution: $y = \frac{1}{\mu}\int \mu f\,dx + \frac{C}{\mu}$
$= e^{\frac{x}{2}}\int e^{-\frac{x}{2}}\cdot\left(\frac{1}{2}e^{x/2}\right)dx + Ce^{\frac{x}{2}}$
$= \frac{x}{2}e^{\frac{x}{2}} + Ce^{\frac{x}{2}}$

Bernoulli's Equation

Standard form: $\frac{dy}{dx} + P(x)y = f(x)y^n$, $n \neq 0, 1$.
Substitution: $y = u^{\frac{1}{1-n}}$, $\frac{dy}{dx} = \frac{1}{1-n}u^{\frac{1}{1-n}-1}\frac{du}{dx}$

3) Solve $x\frac{dy}{dx} + y = \frac{1}{y^2}$

Standard form: $\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}y^{-2}$, $n = -2$

Substitution: $y = u^{\frac{1}{1-n}} = u^{\frac{1}{3}}$

$$\frac{dy}{dx} = \frac{1}{3}u^{-\frac{2}{3}}\frac{du}{dx}$$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x}y^{-2} \Leftrightarrow \frac{1}{3}u^{-\frac{2}{3}}\frac{du}{dx} + \frac{u^{\frac{1}{3}}}{x} = \frac{1}{x}u^{-\frac{2}{3}}$$

$$\Leftrightarrow \frac{du}{dx} + 3\frac{u}{x} = \frac{3}{x}$$

Solve linear Equation: $\frac{du}{dx} + 3\frac{u}{x} = \frac{3}{x}$:

$$\mu = e^{\int \frac{3}{x}dx} = e^{3\ln x} = x^3$$

$$u = \frac{1}{\mu}\int \mu f\,dx + \frac{c}{\mu} = \frac{1}{x^3}\int x^3 \cdot \frac{3}{x}dx + \frac{c}{x^3}$$

$$= \frac{1}{x^3}\int x^2/3\,dx + c/x^3$$

$$= 1 + c/x^3$$

$\therefore \quad y = u^{\frac{1}{3}} = (1 + \frac{c}{x^3})^{1/3}$