Math 2410 # Exam I **Feb 22nd, 2019**

Name: Solutions

Instructions:

- Answer each question to the best of your ability. Show your work or receive no credit.
- All answers must be written clearly. Be sure to erase or cross out any work that you do not want graded. Partial credit can not be awarded unless there is legible work to assess.
- If you require extra space for any answer, you may use the back sides of the exam pages. Please indicate when you have done this so that I do not miss any of your work.
- Calculators are **not** allowed for the exam.

ACADEMIC INTEGRITY AGREEMENT

I certify that all work given in this examination is my own and that, to my knowledge, has not been used by anyone besides myself to their personal advantage. Further, I assert that this examination was taken in accordance with the academic integrity policies of the University of Connecticut.

Signed: Solutions
(full name)

Questions:	1	2	3	4	5	6	**Total**
Score:							

The best angle of attack is the try-angle.

1. (5 points) Consider the IVP

$$\begin{cases} y' = x + y \\ y(0) = 0. \end{cases}$$

Let y be a solution to this IVP, and use Euler's method with step $h = 1$ to approximate $y(3)$.

$y_j = f(x_{j-1}, y_{j-1})\, h + y_{j-1}$ +2

$x_j = x_j + h$

$x_0 = y_0 = 0$

$y_1 = f(0,0) \cdot h + 0 = 0$

$x_1 = 1$

+2

$y_2 = f(x_1, y_1) h + y_1$

$= (1+0) \cdot 1 + 0 = 1$

$x_2 = 2$

$y_3 = f(2,1) h + 1$

$= 2+1 + 1 = 4$

$x_3 = 3$

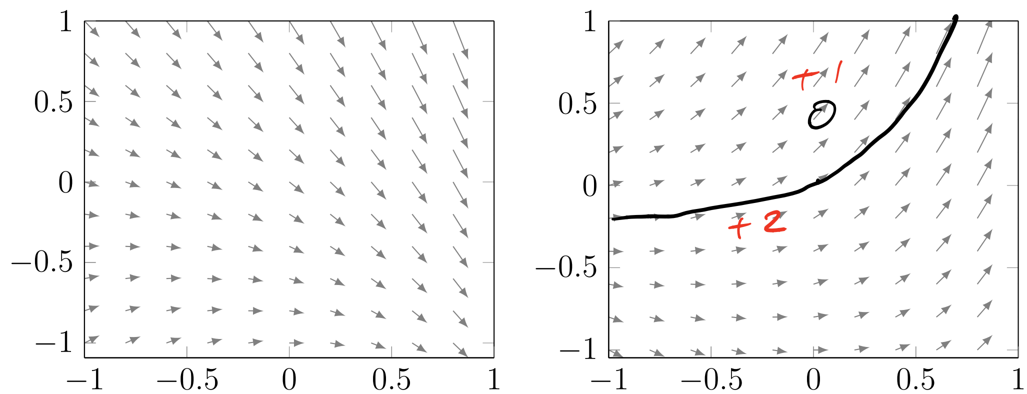
$\rightarrow \underbrace{y(x_3) = y(3) \sim y_3 = 4}_{+1}$

2. (5 points) Below are two direction fields. You will determine which direction field corresponds to the ODE

$$y' = e^x + y.$$

(a) Calculate the slope of the direction field at the point $(0, 0.5)$. Then indicate which of the two direction fields is the correct one by circling the corresponding arrow at $(0, 0.5)$.

(b) On the correct direction field plot, draw a solution curve passing through $(0, 0)$.



(a) $y' = f(0, 0.5) = e^0 + 0.5 = 1.5$

+2

3. (5 points) Consider the ODE

$$-2ydx + (5y - 2x)dy = 0. \tag{1}$$

(a) Show that the equation is exact.

(b) Solve the equation. Your answer may be given as an implicit solution.

Extra credit (2pt): Find the two explicit solutions.

+1 (a) $M = -2y \rightarrow M_y = -2$

$N = 5y - 2x \rightarrow N_x = -2$

+3 (b) $f_x = -2y \rightarrow f = -2xy + g(y)$

$f_y = 5y - 2x \rightarrow f = \frac{5}{2}y^2 - 2xy + h(x)$

$\therefore h(x) = 0$

+1 $\therefore f = -2xy + \frac{5}{2}y^2 = c$

EC

$-2xy + \frac{5}{2}y^2 + c = 0$

$y = \dfrac{+2x \pm \sqrt{4x^2 + c}}{5}$

4. (5 points) Solve the following IVP

$$\begin{cases} y' - y = e^x y^2 \\ y(1) = 2 \end{cases}.$$

Note that the ODE is a Bernoulli's equation. Recall the substitution $y = u^{\frac{1}{1-n}}$.

+1 $n = 2$

$y = u^{-1}$

$\frac{dy}{dx} = -u^{-2}\frac{du}{dx}$

$\therefore \quad -u^{-2}\frac{du}{dx} - u^{-1} = e^x u^{-2}$

+1 $\quad \frac{du}{dx} + u = -e^x$

+1 $\therefore \quad \mu = e^{\int dx} = e^x$

$\therefore \quad u = e^{-x}\int -e^{2x}\,dx + ce^{-x}$

$= -\frac{1}{2}e^x + ce^{-x}$

+1 $\quad y = \frac{1}{-\frac{1}{2}e^x + ce^{-x}}$

+1 $\quad 2 = y(1) = \frac{1}{-\frac{1}{2}e + ce^{-1}} \rightarrow -\frac{1}{2}e + ce^{-1} = \frac{1}{2}$

$c = \frac{1}{2}e + \frac{1}{2}e^2$

5. (5 points) Consider the ODE

$$(x^3 - y^{3k})dx + xy^2 dy = 0$$

where k is to be determined.

(a) Find k so that the given ODE is a homogeneous equation. *Hint:* There is only *one* value for k.

(b) Using this value of k, solve the given ODE.

+2 (a) $t^3x^3 - t^{3k}y^{3k} = t^3(x^3 - t^{3k-3}y^{3k})$ $\therefore k=1$

(b) $y = ux$, $dy = u\,dx + x\,du$

+1 $(x^3 - u^3x^3)dx + x^3u^2(u\,dx + x\,du) = 0$

$x^3dx - \cancel{u^3x^3dx} + \cancel{x^3u^3dx} + x^4u^2du = 0$

+1 $-\frac{1}{x}dx = u^2du$

$-\ln x = \frac{1}{3}u^3 + C = \frac{1}{3}\left(\frac{y}{x}\right)^3 + C$

+1

6. (5 points) Let ϕ be a solution to the ODE

$$y' + y = 0.$$

Prove or disprove that ϕ is a solution to the following ODEs by plugging in ϕ for y:

(a) $(y' + y)(y' - 1) = 0$

(b) $e^{(y'+y)^2} = 1$

(c) $\cos(y' + y) = 0.$

Extra credit (2pt): $y = ce^{-x}$ solves $y' + y = 0$. Find the other family of solutions to

$$(y')^2 + (y-1)y' - y = 0.$$

$\phi' + \phi = 0$

(a) $(\phi' + \phi)(\phi' - 1) = 0 \cdot (\phi' - 1) = 0$

(b) $e^{(\phi'+\phi)^2} = e^0 = 1$

$\Big]$ solution

(c) $\cos(\phi' + \phi) = \cos(0) \neq 0$

EC: $(y')^2 + (y-1)y' - y = (y' + y)(y' - 1) = 0$

$\therefore \quad y' - 1 = 0$

$\therefore \quad y' = 1$

$\therefore \quad y = x + c$