Chapter 2 First Order ODEs

Goal: Study first order ODEs of the forms

$$\frac{dy}{dx} = f(x,y) \quad \text{and} \quad M(x,y)\,dx + N(x,y)\,dy = 0$$

Will learn basic numerical technique and few basic algebraic techniques to solve these ODEs.

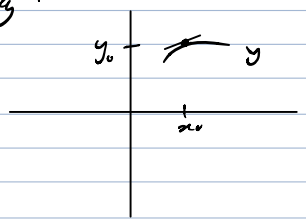
2.1 Solution Curves without a Solution

Goal: Solve $y' = f(x,y)$ qualitatively.

Suppose y is a solution passing through the point (x_0, y_0). Then slope is given as

$$y'(x_0) = f(x_0, y_0).$$

Graphically:



– represents the tangent line of y passing through the point (x_0, y_0) with slope $f(x_0, y_0)$. It is also called a linear element.

So $f(x,y)$ is a "slope function" of solutions to $y' = f(x,y)$.

Note: we can practically do this for any point (x_0, y_0). A rectangular grid with sufficiently many linear elements is called a direction field.

Plan: Use direction fields to qualitatively describe solutions to $y' = f(x,y)$.

Ex

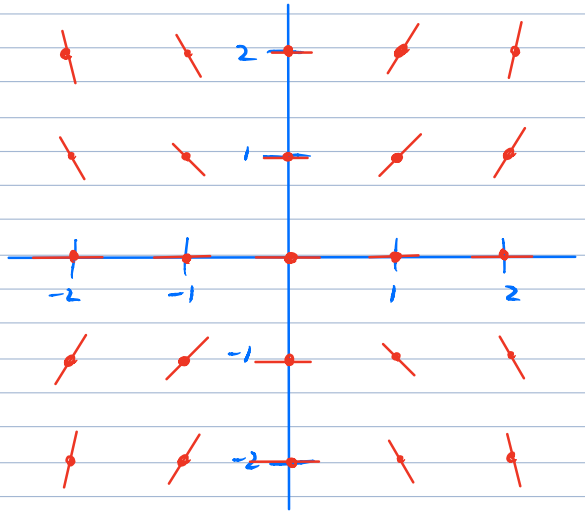
$y' = xy = f(x,y)$

x	y	$y' = f(x,y) = xy$
-2	-2	$-2 \cdot -2 = 4$
-2	-1	$-2 \cdot -1 = 2$
-2	0	$-2 \cdot 0 = 0$
-1	-2	$-1 \cdot -2 = 2$
-1	-1	$-1 \cdot -1 = 1$
-1	0	$-1 \cdot 0 = 0$
0	0	$0 \cdot 0 = 0$
0	1	$0 \cdot 1 = 0$
0	2	$0 \cdot 2 = 0$

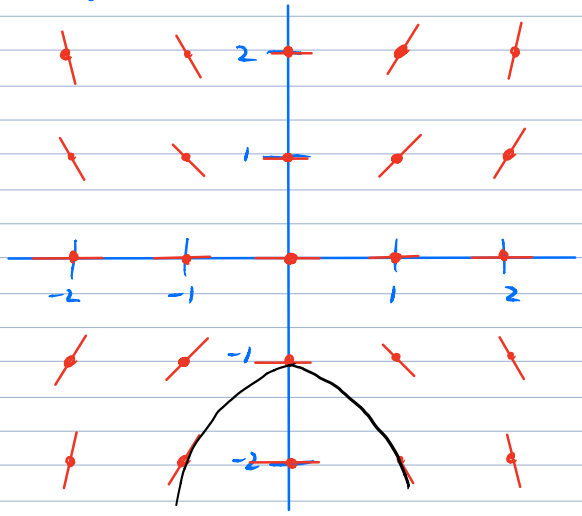
Slopes at (x,y)

. . . . etc. . . . . .

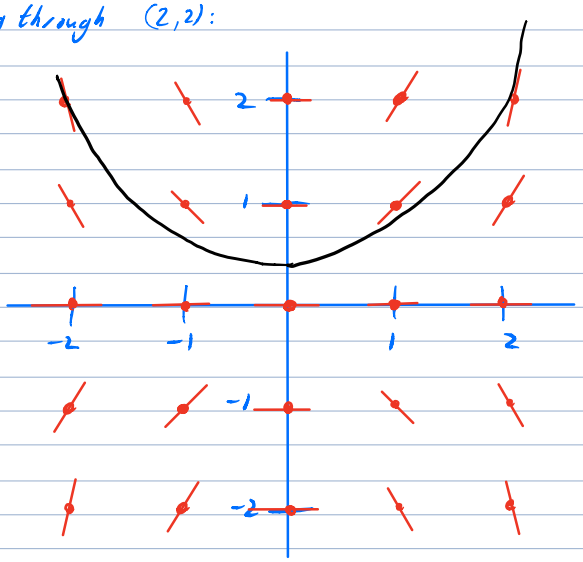
Plot lineal elements



Solution passing through $(0,-1)$:



Solution passing through (2,2):



*More examples done via Web Assign.