Assignment 6

Due: 4/19/18

All relevant work must be shown in your solutions, even if it is not explicitly asked for you to explain.

Problem 1
(3pts)

Solve the boundary-value problem

$$y'' - 2y' + 2y = 0, y(0) = 1, y(\pi) = 1.$$

Problem 2
(3pts)

Solve

$$y'' + y = \cos^2(x)$$

using variation of parameters.

Problem 3
(Extra Credit: 3pts)

Solve the non-linear differential equation

$$(y'')^2 - y^2 = 0$$

by first factoring and then using the tools from Section 4.3.

Problem 1

$$y'' - 2y' + 2y = 0, y(0) = 1, y(\pi) = 1.$$

$m^2 - 2m + 2 = 0 \rightarrow m = \frac{2 \pm \sqrt{4 - 4 \cdot 2}}{2} = \frac{2 \pm \sqrt{-4}}{2} = 1 \pm i$

So $y_c = c_1 e^x \cos x + c_2 e^x \sin x$

$y_c(0) = 1 \rightarrow c_1 = 1$
$y_c(\pi) = 1 \rightarrow -c_1 e^{\pi} = 1$ $\rightarrow c_1 = 1 = -e^{-\pi}$, a contradiction $\rightarrow$ no solution

Problem 2

$$y'' + y = \cos^2(x)$$

$m^2 + 1 = 0 \rightarrow m = \pm i \rightarrow y_c = c_1 \cos x + c_2 \sin x = c_1 y_1 + c_2 y_2$

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = 1$

$u_1' = \frac{-y_2 f(x)}{W} = -\sin x \cos^2 x$ $\quad u_1 = -\int \sin x \cos^2 x = +\int u^2 du = +\frac{u^3}{3} = +\frac{\cos^3 x}{3}$

$u_2' = \cos^3(x) \rightarrow$ $\quad u_2 = \int \cos^3 x = \frac{1}{12}(9 \sin x + \sin 3x)$

Then $y = y_c + y_p = c_1 y_1 + c_2 y_2 + u_1 y_1 + u_2 y_2$

Problem 3

$$(y'')^2 - y^2 = 0$$

$(y'')^2 - y^2 = (y'' - y)(y'' + y) = 0 \rightarrow y'' - y = 0$ or $y'' + y = 0$

$y'' - y = 0 \leftrightarrow m^2 - 1 = 0 \leftrightarrow m = \pm 1 \leftrightarrow y = c_1 e^x + c_2 e^{-x}$

$y'' + y = 0 \rightarrow y = c_3 \cos x + c_4 \sin x$

Se $y = c_1 e^{x} + c_2 e^{-x} + c_3 \cos x + c_4 \sin x$.