Problem 3

$$\tanh(x)\,y' + \sinh(x)\,y = \sinh(x) \longrightarrow y' + \frac{\sinh(x)}{\tanh(x)}\,y = \frac{\sinh(x)}{\tanh(x)}$$

$$\longrightarrow y' + \cosh(x)\,y = \cosh(x)$$

So $P(x) = \cosh(x)$.

The integrating factor:

$$\mu(x) = e^{\int P(x)\,dx} = e^{\int \cosh(x)\,dx} = e^{\sinh(x)}.$$

Hence

$$\frac{d}{dx}(\mu(x)\,y) = \mu(x)\,f(x) = e^{\sinh(x)}\cosh(x)$$

$$\longrightarrow e^{\sinh(x)}\,y = \int e^{\sinh(x)}\cosh(x)\,dx = e^{\sinh(x)} + C \quad \text{by } u\text{-sub.}$$

$$\longrightarrow y = 1 + Ce^{-\sinh(x)}$$

Don't forget the $+C$!

Problem 2

(a) Use $\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$

ODE becomes

$$y' = 2\sinh x \cosh y$$

(b)

$$\operatorname{sech}(y)\,dy = 2\sinh(x)\,dx$$

So by integration:

$$2\tan^{-1}\left(\tanh\left(\frac{y}{2}\right)\right) = 2\cosh(x) + C$$

$$\tanh\frac{y}{2} = \tan(\cosh(x) + C)$$

$$y = 2\tanh^{-1}(\tan(\cosh(x) + C))$$