Assignment 3

Due: 2/8/18

All relevant work must be shown in your solutions, even if it is not explicitly asked for you to explain. You may use pages 1 and 2 to turn in Problem 1. Please use your own paper to turn in your solutions to Problems 2 and 3.

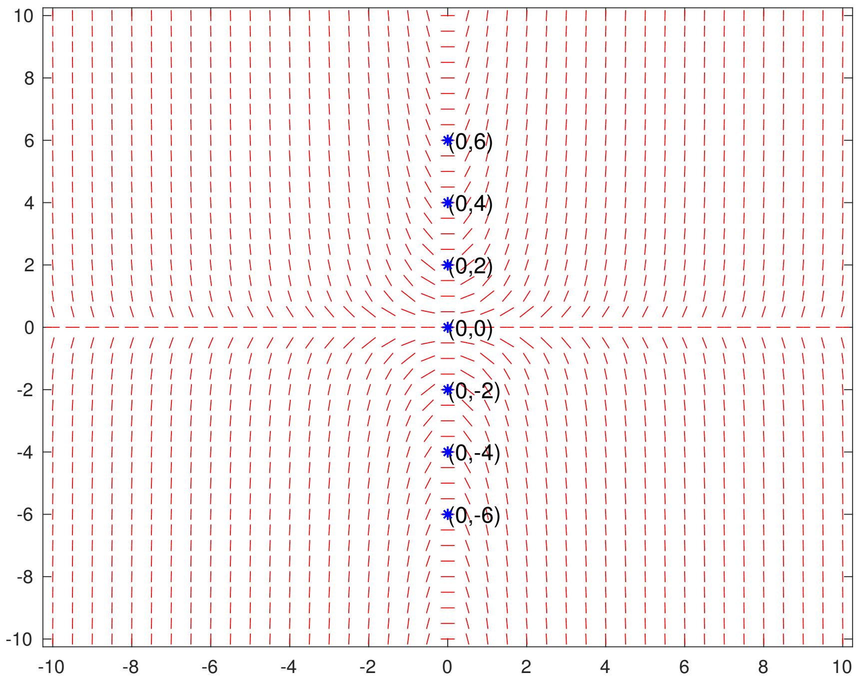
Problem 1

(3pts)

In the following direction fields, plot the solution curve(s) through each of the indicated points on the figures below.

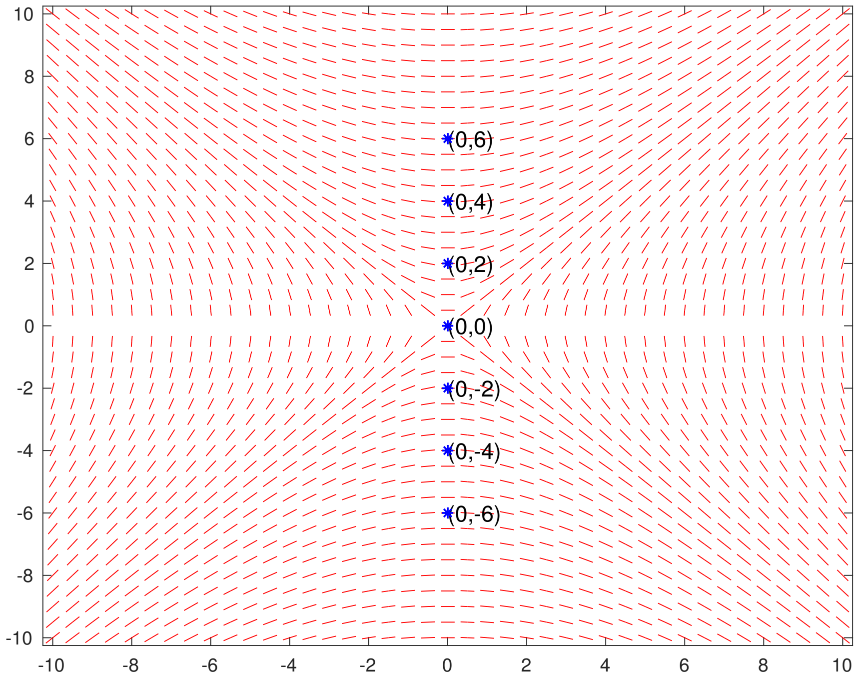
(a) The following is a direction field for the ODE

$$\frac{dy}{dx} = xy.$$



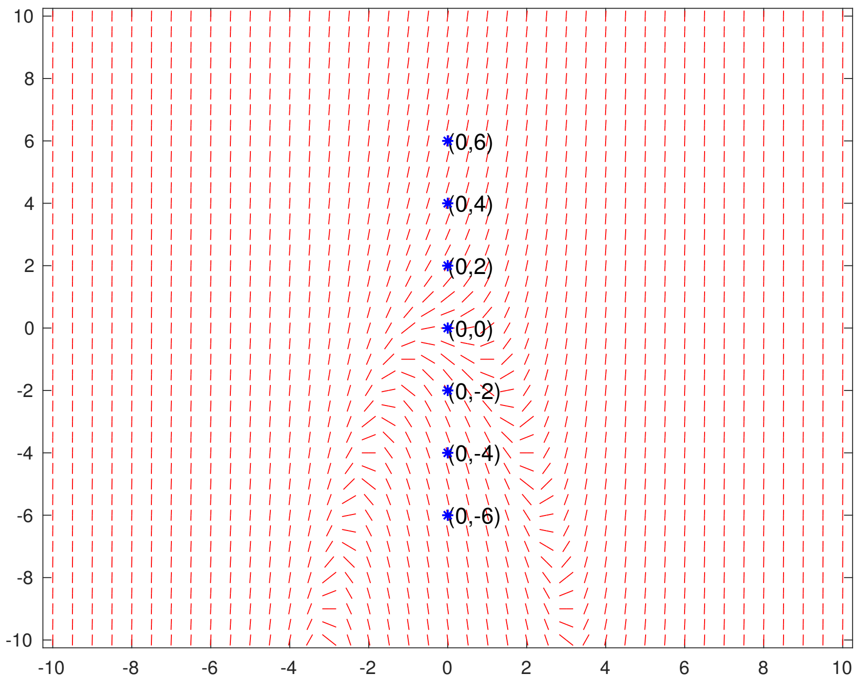
(b) The following is a direction field for the ODE

$$\frac{dy}{dx} = \frac{x}{y}$$



(c) The following is a direction field for the ODE

$$\frac{dy}{dx} = x^2 + y.$$



In Problems 2 and 3, you are expected to use any outside resource to obtain the appropriate hyperbolic trig identities and integrals, e.g., your favorite browser or textbook.[1] *Please cite explicitly the hyperbolic trig identities and integrals you are using. Also cite your resource; e.g., WolframAlpha, Matlab, Mathematica, Maple, Sage, Octave, etc. You may try to do the following problems by hand if you wish.*

Problem 2

(3pts)

Consider the following ODE

$$\frac{dy}{dx} = \sinh(x+y) + \sinh(x-y), \tag{$*$}$$

where $\sinh(z)$ is the hyperbolic sine of z (see below if you are unfamiliar with hyperbolic trig).

(a) Using a single hyperbolic trig identity, rewrite ($*$) by using a separation of variables; i.e., find functions $g = g(x)$ and $h = h(y)$ so that

$$\frac{dy}{dx} = g(x)h(y). \tag{$**$}$$

(b) Rewrite ($**$) in differential form and then use integration to find a solution to ($*$). You may write the solution implicitly or explicitly, but all of your steps should be recorded.

Problem 3

(3pts)

Consider the following Linear ODE

$$\tanh(x)y' + \sinh(x)y = \sinh(x), \tag{$***$}$$

where tanh is the hyperbolic tangent, and sinh is the hyperbolic sine (see below if you are unfamiliar with hyperbolic trig).

(a) Use the method of integrating factors to solve this linear ODE. **State explicitly** what the corresponding $P(x)$ is and what the integrating factor is.

Preliminaries if necessary: The basic hyperbolic trig functions are defined as follows,

$$\begin{aligned}
\sinh(x) &= \frac{e^x - e^{-x}}{2} \\
\cosh(x) &= \frac{e^x + e^{-x}}{2} \\
\tanh(x) &= \frac{\sinh(x)}{\cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}.
\end{aligned}$$

Hence, while they look spooky, hyperbolic trig functions are just rational functions of $e^{\pm x}$. Just as we have the inverse trig functions $\tan^{-1}, \sin^{-1}, \cos^{-1}$, we have the inverse hyperbolic trig functions $\tanh^{-1}, \sinh^{-1}, \cosh^{-1}$ defined in a similar way. So, e.g., to solve $\tanh(y) = x + 1$, we apply $\tanh^{-1}$ to both sides and obtain $y = \tanh^{-1}(x+1)$.

[1] Students (and I) often have the complaint, "I can always just look up an integral if I need to compute one." This is a valid complaint. In these problems, you practice solving ODEs in a more complicated setting, as well as test your preference between doing something *by hand* or *by reference*.

Why hyperbolic trig?: Hyperbolic trig functions arise in various places of application and pure mathematics. For example, they arise in describing catenaries, linear DEs, hyperbolic geometry, special relativity, minimal surfaces, etc. It is a set of tools that should be taught early on, but isn't. If *trigonometry* is a compact geometry (think of the unit circle), then *hyperbolic trigonometry* is a non-compact geometry (think of the hyperbola).