Assignment 1

Problem 1

$$\left(\frac{dy}{dx}\right)^2 - e^{-2y} = 0$$

(a) Highest order derivative is $\frac{dy}{dx}$ $\Rightarrow$ order is 1.

(b) Both

$$\left(\frac{dy}{dx}\right)^2 \quad \text{and} \quad e^{-2y}$$

are nonlinear terms. Hence the ODE is nonlinear.

(c) We check $y(x) = \ln(x+1)$ is a solution first:

$$(y')^2 - e^{-2y} = \left((\ln(x+1))'\right)^2 - e^{-2\ln(x+1)} = (x+1)^{-2} - (x+1)^{-2} = 0.$$

This equation is apparently solved everywhere $y(x) = \ln(x+1)$ and $y'(x) = (x+1)^{-1}$ are defined. This is thus true when $x+1>0$ and hence when $x>-1$. The largest interval is $(-1, +\infty)$.

Problem 2

(a) To find c_1, c_2:

$$x(1) = 1 \Rightarrow c_1 \cos\left(\tfrac{\pi}{2}\cdot 1\right) + c_2 \sin\left(\tfrac{\pi}{2}\cdot 1\right) = c_2 = 1$$

$$x(2) = 2 \Rightarrow c_1 \cos\left(\tfrac{\pi}{2}\cdot 2\right) + 1\cdot \sin\left(\tfrac{\pi}{2}\cdot 2\right) = -c_1 = 2$$

So $c_1 = -2$, $c_2 = 1$ and

$$x(t) = -2\cos\left(\tfrac{\pi}{2}\cdot t\right) + \sin\left(\tfrac{\pi}{2}\cdot t\right)$$

is the particular solution.

Then $x(3) = -2\cos\left(\tfrac{\pi}{2}\cdot 3\right) + \sin\left(\tfrac{\pi}{2}\cdot 3\right) = 0 - 1 = -1$.

Hence $x(3) = -1$ is achieved.

(b) To find c_1, c_2:

$$x(2) = 2 \Rightarrow c_1 \cos\left(\tfrac{\pi}{2}\cdot 2\right) + 1\cdot \sin\left(\tfrac{\pi}{2}\cdot 2\right) = -c_1 = 2$$

$$x(3) = -2 \Rightarrow -2\cos\left(\tfrac{\pi}{2}\cdot 3\right) + c_2 \sin\left(\tfrac{\pi}{2}\cdot 3\right) = -c_2 = -2$$

So $c_1 = -2$, $c_2 = 2$.

Thus

$$x(1) = -2\cos\left(\tfrac{\pi}{2}\cdot 1\right) + 2\sin\left(\tfrac{\pi}{2}\cdot 1\right) = 2$$