Practice Final Exam

Part A: True or False

1. Determine whether the following statements are true or false.

   (a) It is possible to find a function that is continuous everywhere, but not differentiable everywhere.

   (b) A continuous function on $[a, b]$ always has a relative extrema.

   (c) If x is such that $f'(x) = 0$, then x is a critical value of f.

   (d) If $\int_a^b f(x)dx = \int_a^b g(x)dx$, then $f(x) = g(x)$.

   (e) If $f(x)$ is differentiable everywhere, then $f'(x)$ is also differentiable everywhere.

   (f) If x is in the domain of f, but f is not differentiable at x, then x is a critical point of f.

Part B: Logarithms

2. Solve the following equations.

   (a) $\ln(x) = \frac{1}{2}\ln(9) + \ln(5) - \ln(6)$

   (b) $\log_3(9x + 12) = 1 + \log_3(x + 3)$

   (c) $\log_2(16x - 14) + \log_2(x + 8) = 2\log_2(4x)$

   (d) $\log_7(x) = \frac{1}{3}\log_7(8) + \log_7(7) - \log_7(2)$

   (e) $\log_4(10x + 15) = 1 + \log_4(x + 5)$

3. Simplify the following expressions.

   (a) $\log_7(\frac{1}{7^7})$

   (b) $2^{2\log_2(2)}$

   (c) $\log_5(e^{\ln 5})$

   (d) $\log_{12}(3) + \log_{12}(6) + \log_{12}(8)$

Part C: Limits at a Point and Limits at Infinity

4. Evaluate the limit, if it exists. (If it does not exist, write DNE).

   (a) $\lim_{x\to 2} \dfrac{x^2 - x + 6}{x - 2}$

   (b) $\lim_{h\to 0} \dfrac{(1 + h)^2 - 1}{h}$

   (c) $\lim_{x\to -5} \dfrac{x^2 + 6x + 5}{x^2 + 4x - 5}$

   (d) $\lim_{x\to 2} f(x)$, where

$$f(x) = \begin{cases} 8 - x^2 & \text{if } x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$

5. Find all the horizontal asymptotes of the following functions. Give your answer as an equation of a line. If an answer does not exist, write DNE.

   (a) $f(x) = \dfrac{3 + 9e^{-5x}}{12 + 5e^{-5x}}$

   (b) $f(x) = \dfrac{5}{2 + e^{-4x}}$

   (c) $f(x) = \dfrac{x^3 + 3x}{6 - 4x^3}$

Part D: Derivatives

6. Find the derivative of the function $f(x) = \dfrac{3x^2 + 4x^2}{(2x^2 + 4x)^3}$.

7. Find the tangent line of $f(x) = 3xe^x$ at $x = 0$.

8. Let $f(x) = x^2e^{-x}$.
   (a) Find the critical values of $f(x)$.
   (b) Find the intervals on which $f(x)$ increases, and the intervals on which $f(x)$ decreases.
   (c) Find the coordinates of the relative extrema.

9. Let $f(x) = -\frac{x^2}{x+2}$.
   (a) Find the critical values of $f(x)$.
   (b) Find the intervals on which $f(x)$ increases, and the intervals on which $f(x)$ decreases.
   (c) Find the coordinates of the relative extrema.

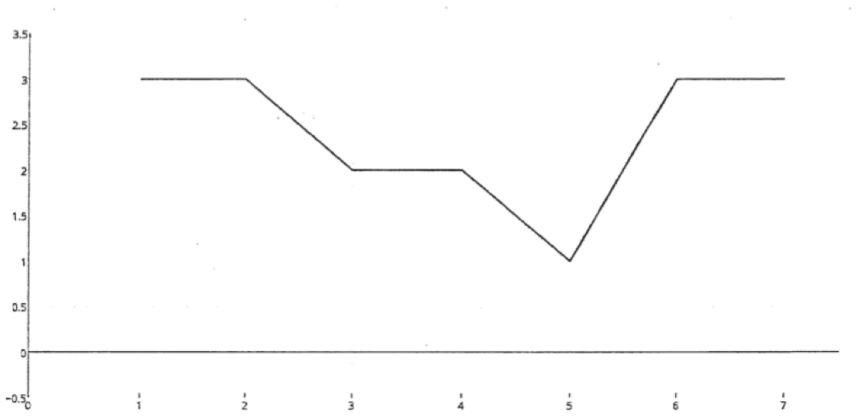
10. Let $f(x) = 2\sqrt[3]{x^5} - 5\sqrt[3]{x^2}$.
    (a) Find the critical values of $f(x)$.
    (b) Find the intervals on which $f(x)$ increases, and the intervals on which $f(x)$ decreases.
    (c) Find the coordinates of the relative extrema.

11. Suppose the function $g(x)$ has a domain of all real numbers except $x = 5$. The second derivative of $g(x)$ is given below.
$$g''(x) = \frac{(x+4)^2(x-3)}{(x-5)^7}$$
    (a) Determine the intervals on which g is concave up, and the intervals on which g is concave down.
    (b) Find the x-coordinates of the inflection points of g.

12. Let $f(x) = 2x^4 + 52x^3 - 14x + 13$.
    (a) Determine the intervals on which f is concave up, and the intervals on which f is concave down.
    (b) Find all inflection points of f.

Part E: Integration

13. Below is a graph of a function f. Find $\displaystyle\int_1^7 f(x)dx$.



14. Find the area between the graph of the function $f(x) = x^4 + e^x + \frac{2}{x}$ and the x-axis on the interval $[1, 2]$.

Part F: Curve Sketching

15. Let $f(x) = e^{-x^2}$.

    (a) Find the domain of f.

    (b) Find the x-intercepts and y-intercepts of f, if they exist.

    (c) Find the **horizontal** asymptotes of f.

    (d) Find f', the critical points of f, the intervals on which f is increasing, and the intervals on which f is decreasing.

    (e) Find f'', the inflection points of f, the intervals on which f is concave up, and the intervals on which f is concave down.

    (f) (Bonus Question) Draw the table with the signs of f' and f'' and the behavior of f.

    (g) Graph f.

    (h) Find the absolute maximum and absolute minimum of f on $(-\infty, \infty)$, if they exist.

16. Let $f(x) = 1 + \ln(x)$.

    (a) Find the domain of f.

    (b) Find the x-intercepts and y-intercepts of f, if they exist.

    (c) Find the **vertical** asymptotes of f.

    (d) Find f', the critical points of f, the intervals on which f is increasing, and the intervals on which f is decreasing.

    (e) Find f'', the inflection points of f, the intervals on which f is concave up, and the intervals on which f is concave down.

    (f) (Bonus Question) Draw the table with the signs of f' and f'' and the behavior of f.

    (g) Graph f.

    (h) Find the absolute maximum and absolute minimum of f on $[1, \infty)$, if they exist.