University of Connecticut
Department of Mathematics

MATH 1071 | PRACTICE EXAM 1 | FALL 2016

NAME: Answer Key

Instructor Name: ______________________ Section: ________

★Reminder: These may not be the only way to solve the problems.

Read This First !

- Please read each question carefully. Show **ALL** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Calculators are allowed, however models TI-89 and above are not permitted.

- **This practice exam is just a guide to prepare for the actual exam. It may take more or less time to take than the actual exam. Questions on the actual exam may or may not be of the same type, or nature. Do not prepare only by taking this exam. You should also review class notes, WebAssign, and in class quizzes.**

1. The revenue and cost functions for a product are given below. The revenue and cost are given in dollars and x represents the number of units.

$$\text{Revenue: } R(x) = -0.5x^2 + 20x$$

$$\text{Cost: } C(x) = 10x + 18$$

(a) How many units must be sold to maximize the revenue?

$$x = \frac{-b}{2a} = \frac{-20}{2(-0.5)} = \frac{-20}{-1} = 20$$

$$\boxed{20 \text{ units}}$$

(b) At what production level(s) will the company break even?

$P(x) = 0$ so $R - C = 0$

$$-0.5x^2 + 20x - (10x + 18) = 0$$

$$-0.5x^2 + 20x - 10x - 18 = 0$$

$$-0.5x^2 + 10x - 18 = 0$$

Quad. Formula:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-10 \pm \sqrt{100 - 4(-0.5)(-18)}}{2(-0.5)}$$

$$= \frac{-10 \pm \sqrt{100 - 36}}{-1} = \frac{-10 \pm \sqrt{64}}{-1} = 10 \pm 8$$

$$\boxed{x = 2, 18}$$

2. The supply equation for a particular product is known to be $p = .15x + 10$.

   (a) Consumers are willing to buy 15 units of these products when the price is set at \$10. If the price is increased to \$20, the consumers will buy 5 units less. Find the demand equation for the product, assuming a linear relationship between price, p, and quantity, x.

$x = 15 \Rightarrow p = 10$

$x = 15 - 5 \Rightarrow p = 20$

$x = 10$

Two points: $(15, 10)$, $(10, 20)$

Slope: $m = \dfrac{20-10}{10-15} = \dfrac{10}{-5} = -2$

Point-slope form:

$p - 10 = -2(x - 15)$

$p - 10 = -2x + 30$

$\boxed{p = -2x + 40}$

OR

Slope-intercept:

$y = mx + b$

$10 = -2(15) + b$

$10 = -30 + b$

$40 = b$

$\boxed{y = -2x + 40}$

   (b) Find the equilibrium point. Round to 3 decimal places.

From statement: Supply is $p = .15x + 10$

So $.15x + 10 = -2x + 40$

$2.15x + 10 = 40$

$2.15x = 30$

$x = \dfrac{30}{2.15} = 13.953$,

When $x = 13.953$

$p = .15(13.953) + 10 = 12.093$

$\boxed{(13.953, 12.093)}$

3. It costs \$100 to rent a table at a local flea market and it costs \$5.50 for the materials that are used to assemble each of the handmade frames that you would like to sell there. You have decided to charge \$15 for each frame.

(a) Find the linear cost function for the frames.

Variable: $5.5x$

Fixed: 100

So $\boxed{C = 5.5x + 100}$

(b) Find the linear revenue function for the frames.

$p = 15$ so

$\boxed{R = 15x}$

(c) Find the profit function for the frames.

$P = R - C$

$$P = 15x - (5.5x + 100)$$
$$= 15x - 5.5x - 100$$
$$\boxed{P = 9.5x - 100}$$

(d) How many frames would you need to sell to break-even?

$P(x) = 0$

So $9.5x - 100 = 0$

$$9.5x = 100$$
$$x = \frac{100}{9.5} = 10.526$$

$\boxed{\text{So 11 Frames}}$

4. Solve for x in the following expressions. Give an exact answer.

(a) $\ln(x) = 3\ln(2) - \ln(4) + 2\ln(5)$

$\ln(x) = \ln(2^3) - \ln(4) + \ln(5^2)$

$= \ln(8) - \ln(4) + \ln(25)$

$= \ln\left(\frac{8 \cdot 25}{4}\right) = \ln(50)$

$\Rightarrow \ln(x) = \ln(50)$

$\boxed{x = 50}$

(b) $7 \cdot (5^x) + 4 = 19$

$7 \cdot 5^x = 15$

$5^x = 15/7$

$\log_5(5^x) = \log_5(15/7)$

$\boxed{x = \log_5(15/7)}$

OR

$5^x = 15/7$

$\ln(5^x) = \ln(15/7)$

$x\ln(5) = \ln(15/7)$

$\boxed{x = \frac{\ln(15/7)}{\ln(5)}}$

(c) $16^{3x+1} = 64^{x-5}$

$(4^2)^{3x+1} = (4^3)^{x-5}$

$4^{6x+2} = 4^{3x-15}$

$\Rightarrow 6x+2 = 3x-15$

$3x = -17$

$\boxed{x = \frac{-17}{3}}$

(d) $3^{-x^2+2x} = 3^1$

So $-x^2 + 2x = 1$

$-x^2 + 2x - 1 = 0$

$x^2 - 2x + 1 = 0$

$(x-1)(x-1) = 0$

$\boxed{x = 1}$

5. Evaluate the following logarithm and simplify are far as possible:

(a) $9^{2\log_9(4)+\log_9(2)}$

$9^{\log_9(4^2)+\log_9(2)} = 9^{\log_9(16\cdot 2)} = 16 \cdot 2 = \boxed{32}$

(b) $\log_4(5 \cdot 4^3)$

$\log_4(5) + \log_4(4^3) = \boxed{\log_4(5) + 3}$

6. An account earns an annual rate of 7% and is compounded quarterly. Determine the amount of money you would need to set aside to have \$20,000 after 15 years.

$F = 20000$

$t = 15$

$m = 4$

$r = 0.07$

$$P = \frac{F}{\left(1+\frac{r}{m}\right)^{mt}} = \frac{20000}{\left(1+\frac{0.07}{4}\right)^{4\cdot 15}}$$

$$= \$7062.61$$

7. Given the function $f(x) = \frac{1}{x+2}$, compute the following where $h \neq 0$.

$$\frac{f(x+h) - f(x)}{h}$$

$f(x+h) = \frac{1}{x+h+2}$

$f(x) = \frac{1}{x+2}$

$$f(x+h) - f(x) = \frac{1}{x+h+2} - \frac{1}{x+2}$$

$$= \frac{(x+2)}{(x+2)(x+h+2)} - \frac{(x+h+2)}{(x+2)(x+h+2)}$$

$$= \frac{x+2-(x+h+2)}{(x+2)(x+h+2)} = \frac{\cancel{x}+\cancel{2}-\cancel{x}-h-\cancel{2}}{(x+2)(x+h+2)}$$

$$= \frac{-h}{(x+2)(x+h+2)}$$

So $\frac{f(x+h)-f(x)}{h} = \frac{\frac{-\cancel{h}1}{(x+2)(x+h+2)}}{\cancel{h}} = \boxed{\frac{-1}{(x+2)(x+h+2)}}$

8. Compute:

$$\lim_{x\to -2} \frac{x^2 - 2x - 8}{x^2 - 4}$$

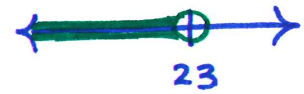
$$\lim_{x\to -2} \frac{x^2-2x-8}{x^2-4} = \lim_{x\to -2} \frac{(x-4)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}} = \frac{-2-4}{-2-2} = \frac{-6}{-4}$$

$$= \boxed{\frac{3}{2}}$$

9. In interval notation, state the domain of the function below:

(a) $f(x) = \frac{x^3}{\sqrt{23 - x}}$

$23 - x > 0$
$+x \qquad +x$
$23 > x$



$\boxed{(-\infty, 23)}$

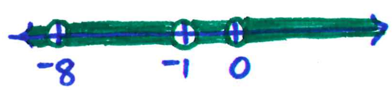
(b) $f(x) = \frac{1}{x^3+9x^2+8x}$ ← $\neq 0$

$x^3 + 9x^2 + 8x = 0$

$x(x^2 + 9x + 8) = 0$

$x(x+8)(x+1) = 0$

$x \neq 0, \ x \neq -8, \ x \neq -1$



$\boxed{(-\infty, -8) \cup (-8, -1) \cup (-1, 0) \cup (0, \infty)}$

(c) $f(x) = \frac{\sqrt{4x+2}}{x^2}$

$4x + 2 \geq 0$ and $x^2 \neq 0$
So $x \neq 0$.

$4x \geq -2$

$x \geq -\frac{2}{4}$

$x \geq -\frac{1}{2}$

$\boxed{[-\frac{1}{2}, 0) \cup (0, \infty)}$

10. For the following function

$$f(x) = \begin{cases} 1 & \text{if } -5 < x < -1, \\ 3x-1 & \text{if } -1 \le x < 5, \\ 5-x & \text{if } 6 \le x < 10 \end{cases}$$

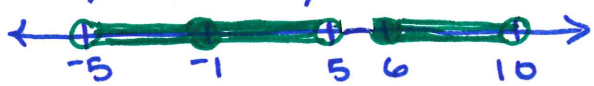
(a) Find the domain.

1, $3x-1$, $5-x$ are defined everywhere.

"If statements" tell me

$(-5,-1) \cup [-1,5) \cup [6,10)$

So



$\boxed{(-5,5) \cup [6,10)}$

(b) In interval notation, state the values of x for which $f(x)$ is continuous.

1, $3x-1$, $5-x$ are cont. everywhere.

So check at "rule switches"

$x=-1$:

$\lim_{x \to -1^-} f(x) = \lim_{x \to -1^-} 1 = 1$

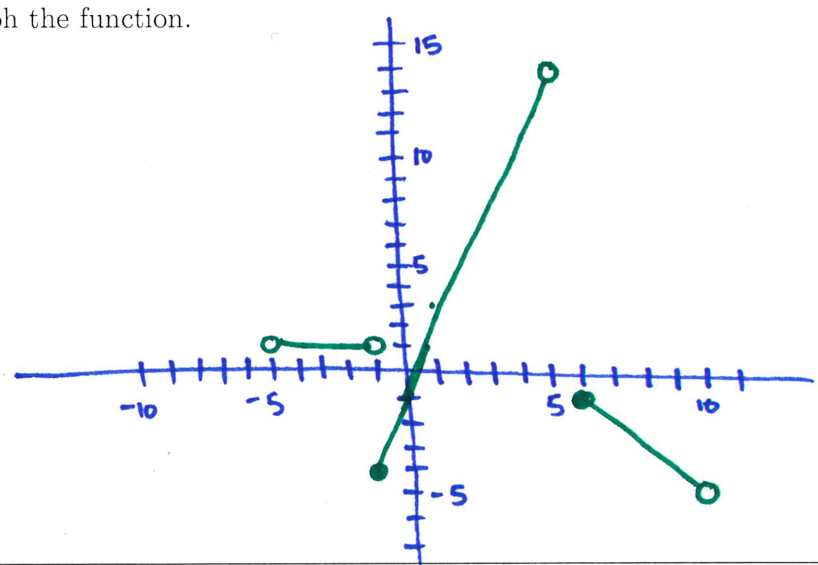
$\lim_{x \to -1^+} f(x) = \lim_{x \to -1^+} 3x-1 = -3-1 = -4$

$1 \neq -4$, discont. at $x=-1$.

between $x=5$ and $x=6$, function does not exist. So continuous on:

$\boxed{(-5,-1) \cup (-1,5) \cup (6,10)}$

(c) Graph the function.



11. A package shipping company has different rate for packages ship within United States. For packages weigh under 1lb, they charge a flat rate of \$12. For each additional pound over 1lb (up to 5lb), they charge \$2 per pound. For each additional pound over 5lb, they charge \$5 per pound. Write down the piece-wise function that calculates the amount of shipping fee S, in relationship to the weight of the packages x.

\$12 flat rate + \$2 per extra lb.

$$S = \begin{cases} 12 & \text{if } x \le 1 \\ 12 + 2(x-1) & \text{if } 1 < x \le 5 \\ \underset{\substack{\uparrow \\ \text{flat rate}}}{12} + \underset{\substack{\uparrow \\ \$2 \text{ per lb for } 4 \text{ lbs}}}{2(5-1)} + \underset{\substack{\uparrow \\ \$5 \text{ per lb for each lb over } 5.}}{5(x-5)} & \text{if } 5 < x. \end{cases}$$

12. Given the funtion $f(x) = \dfrac{x^2 - 14x + 49}{x^2 - 8x + 7}$

(a) Is the function $f(x)$ defined at $x = 7$?

$$x = 7 \Rightarrow \frac{7^2 - 14(7) + 49}{7^2 - 8(7) + 7} = \frac{49 - 98 + 49}{49 - 56 + 7} = \frac{0}{0}$$

$\boxed{\text{No.}}$

(b) Does the limit of $f(x)$ as x approaches 7 exist? If yes, specify the value. If no, explain why it does not exist.

$$\lim_{x \to 7} \frac{x^2 - 14x + 49}{x^2 - 8x + 7} = \lim_{x \to 7} \frac{(x-7)(x-7)}{(x-7)(x-1)} = \lim_{x \to 7} \frac{x-7}{x-1}$$

$$= \frac{0}{6} = 0.$$

$\boxed{\text{Yes, the limit is } 0}$

13. Evaluate

$$\lim_{x\to 5}\frac{3x}{|x-5|}$$

Need left & right limits.

$$\lim_{x\to 5^-}\frac{3x}{|x-5|} = \frac{\oplus 15}{\oplus 0} = +\infty$$

$$\lim_{x\to 5^+}\frac{3x}{|x-5|} = \frac{\oplus 15}{\oplus 0} = +\infty$$

OR recognize $|x-5| \to 0$, always positive, so $\frac{\oplus}{\oplus}$

$$\boxed{\lim_{x\to 5}\frac{3x}{|x-5|} = +\infty}$$

14. Given the function $f(x) = \dfrac{8}{x-6}$

(a) Evaluate

$$\lim_{x\to 6^-} f(x)$$

constant

$$\lim_{x\to 6^-}\frac{8}{x-6} = \frac{\oplus}{\ominus} = \ominus$$

approaches 0

Since approaching from left, $x-6<0$ so $\ominus$

$$\boxed{\text{So } \lim_{x\to 6^-}\frac{8}{x-6} = -\infty}$$

(b) Evaluate

$$\lim_{x\to 6^+} f(x)$$

$$\lim_{x\to 6^+}\frac{8}{x-6} = \frac{\oplus}{\oplus} = \oplus$$

$$\boxed{\text{So } \lim_{x\to 6^+}\frac{8}{x-6} = +\infty}$$

Since approaching from right, $x-6>0$ so $\oplus$

15. Find the point-slope equation of the line that passes through the point $(2,3)$, which is perpendicular to the line $3x+2y=4$, and graph it.

Perpendicular: $m_1 = -\frac{1}{m_2}$

Slope of $3x+2y=4$?

$2y = 4-3x$

$y = 2-\frac{3}{2}x$

Slope is $-\frac{3}{2}$.

So slope of our line is

$m = -\frac{1}{-3/2} = +\frac{2}{3}$

Point-slope: $m=\frac{2}{3}$, point: $(2,3)$

$$\boxed{y-3=\frac{2}{3}(x-2)}$$

16. Find the point-slope equation of the line that passes through the point $(1,5)$, which is parallel to the line $y=2x+1$, and graph it.

Parallel: $m_1 = m_2$

Slope of $y=2x+1$?

Slope is 2.

So slope of our line is $m=2$.

Point-slope; $m=2$, point: $(1,5)$

$$\boxed{y-5=2(x-1)}$$

17. Find the point-slope equation of the line that passes through the points $(2,3)$ and $(1,5)$, and graph it.

$$m = \frac{5-3}{1-2} = \frac{2}{-1} = -2$$

Point-slope: $m=-2$, point: $(2,3)$

$$\boxed{y-3=-2(x-2)}$$

or

point: $(1,5)$

$$\boxed{y-5=-2(x-1)}$$

18. Determine the roots of the polynomial $2x^5-32x$.

$$2x^5-32x = 2x(x^4-16) = 0$$

$x=0$

$x^4-16=0$

$x^4=16$

$x = \pm\sqrt[4]{16} = \boxed{\pm 2}$

19. Simplify completely:

$$\frac{x^2-9}{x^2+2x-3}$$

$$\frac{x^2-9}{x^2+2x-3} = \frac{(x-3)(x+3)}{(x+3)(x-1)} = \boxed{\frac{x-3}{x-1}}$$

20. Simplify completely:

$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h}$$

$$\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{\frac{x}{x(x+h)}-\frac{x+h}{x(x+h)}}{h} = \frac{\left(\frac{x-(x+h)}{x(x+h)}\right)}{h} = \frac{\left(\frac{x-x-h}{x(x+h)}\right)}{h}$$

$$= \frac{\left(\frac{-h}{x(x+h)}\right)}{\frac{h}{1}} = \frac{-\cancel{h}^{1}}{x(x+h)}\cdot\frac{1}{\cancel{h}} = \boxed{\frac{-1}{x(x+h)}}$$

21. Solve the following inequality, and write your answer in interval notation:

$$0 \leq x+3 < 9$$

$$-3 \qquad -3 \qquad -3$$

$$-3 \leq x < 6$$

$$\boxed{[-3,6)}$$

22. Solve the following inequality, and write your answer in interval notation:

$$9 \le |2x - 1|$$

$|2x-1| \ge 9$ so

$2x - 1 \ge 9$ or $2x - 1 \le -9$

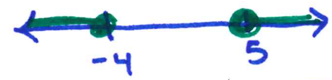
$+1 \quad +1 \qquad +1 \quad +1$

$2x \ge 10 \qquad 2x \le -8$

$\div 2 \quad \div 2 \qquad \div 2 \quad \div 2$

$x \ge 5 \qquad x \le -4$

$x \ge 5$ or



$\boxed{(-\infty, -4] \cup [5, \infty)}$

23. Solve the following inequality

$$|2x - 1| \le 9$$

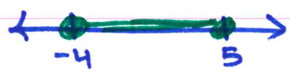
So $-9 \le 2x - 1 \le 9$

$+1 \quad +1 \quad +1$

$-8 \le 2x \le 10$

$\div 2 \quad \div 2 \quad \div 2$

$-4 \le x \le 5$



$\boxed{[-4, 5]}$

24. Simplify. Write your answers without roots or negative exponents.

$$\sqrt{32a^{10}b^{15}}$$

$$\sqrt{32a^{10}b^{15}} = (32a^{10}b^{15})^{1/2} = 32^{1/2} a^{10/2} b^{15/2} = \boxed{32^{1/2} a^5 b^{15/2}}$$

25. Simplify. Write your answers without roots or negative exponents.

$$\left(\frac{x^2y^{-5}}{x^{-2}y^{-3}}\right)^{-3}$$

$$\left(\frac{x^2y^{-5}}{x^{-2}y^{-3}}\right)^{-3} = \frac{x^{-6}y^{15}}{x^6y^9} = \frac{y^{15}}{x^6x^6y^9} = \frac{y^{15-9}}{x^{6+6}} = \boxed{\frac{y^6}{x^{12}}}$$