Math 1071 Quiz 8

Problem 1. Let $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 6x - 12$. Determine where f is increasing or decreasing and identify the x values for any relative min or max values of f.

Solution 1. We find $f'(x) = x^2 - 5x + 6 = (x-3)(x-2)$ and so our critical points are $x = 2$ and $x = 3$. Therefore $f'(x) = 0$ only for $x = 2$ and $x = 3$ and we note it is continuous everywhere. Therefore, by the constant sign theorem, we consider the intervals $(-\infty, 2), (2, 3), (3, \infty)$. We find that, since 0 is in $(-\infty, 2)$ and $f'(0) = (0-2)(0-3) = 6 > 0$ that, by the constant sign theorem, $f'(x) > 0$ on $(-\infty, 2)$. Next, 2.5 is in $(2, 3)$ and so $f'(2.5) = (2.5-2)(2.5-3) = (0.5)(-0.5) < 0$ and so $f'(x) < 0$ on $(2, 3)$. Lastly, since 4 is in $(3, \infty)$ and $f'(4) = (4-2)(4-3) = 2 > 0$, we get that $f'(x) > 0$ on $(3, \infty)$.

With this data we may conclude that f is increasing on $(-\infty, 2) \cup (3, \infty)$ since this is where f' is positive. Secondly, f is decreasing on $(2, 3)$ since this is where f' is negative.

Lastly, using the first derivative test, we find that since f' is positive on $(-\infty, 2)$ and negative on $(2, 3)$ that $f(2)$ is a relative maximum and so $x = 2$ is an x-coordinate for a relative maximum. Next, since f' is negative on $(2, 3)$ and positive on $(3, \infty)$, we conclude that $f(3)$ is a relative minimum and so $x = 3$ is the x-coordinate.

Problem 2. Let $g(x) = x^3 + x$ and find the inflection points of g.

Solution 2. To do this problem we must first find $g''(x)$. We get $g''(x) = 6x$. Recall that in order for a point x to be an inflection point, we must have $g''(x) = 0$ and, since $g''(x) = 6x = 0$ only when $x = 0$, $x = 0$ is the only point we need to care about in terms of finding the inflection points of g. So we now test for whether or not $x = 0$ is an inflection point for g by determining where g is concave up and down.

We use the constant sign theorem. Note that the only two intervals of interest are $(-\infty, 0)$ and $(0, \infty)$ since this is precisely where g'' is continuous and non-zero. Note that -1 is in $(-\infty, 0)$ and by $g''(-1) = -6 < 0$ we may conclude that g is concave down on $(-\infty, 0)$. Furthermore, since 1 is in $(0, \infty)$ and $g''(1) = 6 > 0$, we may conclude that g is concave up on $(0, \infty)$. This is enough to conclude $x = 0$ is an inflection point of g by comparing this data to the definition of an inflection point.