Problem 1. Compute derivatives of the following expressions.

1. $x^{-\frac{1}{2}} + e^{\pi} + 3$
2. $e^x + x^2 + \ln x^3 + \log(x)$.
3. $(x^2 + \sqrt{x})e^x$
4. $5^x + x$.
5. $\ln(x + e^x)$
6. $e^{\ln(x)+x^{-2}}$
7. $\frac{e^x+x^{3.4}}{\ln x}$
8. $\frac{(x+\ln(x^2+1))(x^2+1)}{\sqrt{x+e^x}}$

Problem 2. Use the limit definition to compute the derivative of the following functions.

1. $f(x) = x^2$
2. $g(x) = x + a$
3. $F(x) = \frac{1}{x+a}$
4. $G(x) = \frac{1}{x^2}$

Problem 3.

1. Compute the following average rate of changes over the indicated intervals
   (a) $f(x) = x^2$ over $[0, 1]$
   (b) $g(x) = \ln x$ over $[1, e]$
   (c) $F(x) = x^3 + x$ over $[3, 4]$
   (d) $G(x) = \sqrt{x}$ as x changes from 3 to 5
2. Compute the instantaneous rate of changes of the following functions at the indicated points
   (a) $f(x) = x^2$ at $x = 0$
   (b) $g(x) = \ln x$ at $x = 1$
   (c) $F(x) = x^3 + x$ at $x = 0$
   (d) $G(x) = \sqrt{x}$ at $x = 4$
3. Compute tangent lines to the graphs of the following functions at the indicated point.
   (a) $f(x) = x^2$ at $x = 0$
   (b) $g(x) = \ln x$ at $x = 1$
   (c) $F(x) = x^3 + x$ at $(0, F(0))$
   (d) $G(x) = \sqrt{x}$ at $x = 4$

Problem 4. Make sure you understand how to determine where a function is differentiable, continuous, or has a limit given its graph. Further, make sure you understand how to compute left hand, right hand, and limits given a graph. Also, be able to determine where a graph has positive slope, negative slope, zero slope. Recall:

- A function is not differentiable at a point its graph has a very sharp corner or where a function is discontinuous.

- A function is not continuous (i.e., discontinuous) at points where the function is unbounded, where the left hand and right limits do not agree, where the limit does not agree with the function, or the function isn't defined.
- A function does not have a limit when it is unbounded or its left hand and right hand limits disagree.

Problem 5. Compute the following limits or explain why they do not exist.

1. $\lim_{x\to 0}(x^3+x^2+x+1)$
2. $\lim_{x\to -1}\frac{x^2+2x+1}{x+1}$
3. $\lim_{x\to -1}\frac{x^2-2x+1}{x+1}$
4. $\lim_{x\to 3}\frac{x^2-9}{x+3}$
5. $\lim_{x\to 3}\frac{1}{x-3}$
6. Let
$$f(x)=\begin{cases}x & x\geq 0\\ -x+1 & x<0\end{cases}$$
and compute $\lim_{x\to 0} f(x)$ if it exists.
7. Let
$$f(x)=\begin{cases}\sqrt{x+1} & x\geq 0\\ -x+1 & x<0\end{cases}$$
and compute $\lim_{x\to 0} f(x)$ if it exists.

Note: When you are said to explain your answer if the limit does not exist, you are basically being asked to explicitly compute the left hand and right hand limits and show that they do not exist. This is not the same as just saying "left hand limit $\neq$ right hand limit". Be explicit! The same is to be said for similar problems using graphs.

Problem 6. Determine where the following functions are discontinuous if anywhere

1. $f(x)=\frac{x^2-1}{x+1}$
2. $g(x)=\frac{x}{x^2+2x+1}$
3.
$$f(x)=\begin{cases}\sqrt{x+1} & x\geq 0\\ -x+1 & x<0\end{cases}$$
4.
$$f(x)=\begin{cases}x^2+x & x\geq 2\\ \sqrt{x^2+1} & x<0\end{cases}$$
5.
$$f(x)=\begin{cases}\ln(x+1) & x\geq 0\\ \frac{1}{-x+1} & x<0\end{cases}$$

Problem 7.
1. Suppose you want to earn 50 dollars after 2 years on an account that earns 10% annually and is compounded quarterly. How long will it take to do so?
2. Suppose you want to earn 50 dollars after 2 years on an account that earns 10% annually and is compounded continuously. How long will it take to do so?

3. Suppose you invest 100 dollars into an account that yields 5% annually. How much will be in the account after 3 years if the account is compounded monthly. How much will be in the account after 3 years if the account is compounded continuously?

Problem 8. Find the maximum or minimum values of the following quadratics.

1. $x^2 + x$
2. $-x^2 - 2x + 1$
3. $2x^2$
4. $10x^2 + 2x - 1$

Problem 9. Here I will just outright give you the functions needed to do computations. In the exam, you might be faced with a "word problem" where you must parse/extract the information necessary for finding the functions.

1. Suppose the cost function of some product is $C(x) = 2x + 1$ and the revenue function is $R(x) = -x^2 + 1$.
   (a) Find the maximum revenue.
   (b) Find the profit function.
   (c) Find the maximum profit.
   (d) Find the break even point.

1. Suppose the cost function of some product is $C(x) = 2x^2 + 1$ and the revenue function is $R(x) = -3x^2 + 1$.
   (a) Find the minimum cost.
   (b) Find the maximum revenue.
   (c) Find the profit function.
   (d) Find the maximum profit.
   (e) Find the break even point.

Problem 10. Solving equations using logarithms and exponents. In the following problems, solve for x.

1. $5^{2x} = 25^{3x+1}$
2. $e^3 = e^{x^2+1}$
3. $4^{3x} = \left(\frac{1}{64}\right)^x$
4. $\log(x + 2) - \log(2) = 4$
5. $\log_2(x^2) + \log_2(4) = 3$
6. $2 \ln x = 10 \ln 4$