3.2 Rates of Change

Definitions

Definition 1 (Average Rate of Change). The average rate of change of $y = f(x)$ with respect to x from a to b is

$$\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{f(b) - f(a)}{b - a}.$$

Definition 2 (Secant line?).

Definition 3 (Velocity?).

Definition 4 (Instantaneous Rate of Change). Given a function $y = f(x)$, the instantaneous rate of change of y with respect to x at $x = c$ is given by

$$\lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$$

if this limit exists.

Definition 5 (Tangent Line). The tangent line to the graph of $y = f(x)$ at $x = c$ is the line through the point $(c, f(c))$ with slope

$$m_{tan}(c) = \lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$$

provided the limit exists.

Concepts

Average rate of change given a table

cf. lecture

Average rate of change given a graph

cf. lecture

Average rate of change given a function

Example. Let $f(x) = x^3 - 1$. Find the average rate of change over the interval $[0, 2]$.

Example 1. Let $g(x) = \sqrt{x + 1}$. Find the average rate of change of $f(x)$ as x changes from 3 to 8.

Instantaneous rate of change

Example. Let $f(x) = x^2 + x$ and find the instantaneous rate of change at $x = 1$.

Example. Let $g(x) = 2x^2 - 2x + 1$ and find the instantaneous rate of change at $x = 0$.

Slopes of Graphs

cf. lecture

Tangent line and its equation

Example. Let $f(x) = x^2$ and find the equation of the tangent line through the point $(3, f(3))$.

Necessary Concepts to Know

Here is a minimum list of concepts you should know.

1. Average rate of change given a table

2. Average rate of change given a graph

3. Average rate of change given a function

4. Instantaneous rate of change

5. Slopes of graphs

6. Tangent line and its equation