1.1 Functions

Definitions

Definition 1 (Function)**.** *Let D and R be two nonempty sets. A function f from D to R is a rule that assigns to* ***each*** *element x in D* ***one and only one*** *element $y = f(x)$ in R.*

Definition 2 (Domain, Codomain, Range)**.** *Let f be a function from D to R and let Y be the set of all outputs of f. Then*

1. *D is called the domain.*
2. *R is called the codomain.*
3. *Y is called the range.*

Example 1. Let $f(x) = 2x$ and $g(x) = 2 - x^3$. Find $f(3)$ and $g(2)/f(5)$.

Conventions

Domains

When the domain of f is not explicitly stated, we assume the domain is the set of values x such that $f(x)$ makes sense and is a real number.

Example 2. What is the domain of $f(x) = 1/x$?

Interval notation

(a, b)	$a < x < b$
$(a, b]$	$a < x \leq b$
$[a, b)$	$a \leq x < b$
$[a, b]$	$a \leq x \leq b$
$(-\infty, b)$	$-\infty < x < b$
$(-\infty, b]$	$-\infty < x \leq b$
(a, ∞)	$a < x < \infty$
$[a, \infty)$	$a \leq x < \infty$
$(-\infty, \infty)$	$-\infty < x < \infty$

Union, Intersection Notation

Let A and B be any two sets. Then,

1. Union: $A \cup B =$ all elements in **either** A **or** B.
2. Intersection: $A \cap B =$ all elements in **both** A **and** B.

Example 3.

1. $(-1, 1] \cup [1, 2) = (-1, 2)$
2. $(-1, 1) \cap (0, 1) = (0, 1)$.

Example 4.

1. Domain of $f(x) = 1/x$ is all x such that $x \neq 0$; i.e., $(-\infty, 0) \cup (0, \infty)$.
2. Find the domain of $f(x) = \sqrt{}(2x - 4)$.
3. Find the domain of $f(x) = \frac{x^2}{(x^2-2x+1)}$.
4. Find the domain of $f(x) = \frac{1}{x^2+1}$.

Piecewise Function

Given by example:

Example 5. Let f be the function defined by

$$f(x) = \begin{cases} x+1 & x \geq 0 \\ -2x & x < 0 \end{cases}$$

Then by this notation we mean that when $x \geq 0$, $f(x) = x + 1$ and when $x < 0$, then $f(x) = -2x$; e.g., $f(3) = 3 + 1 = 4$ and $f(-5) = -2(-5) = 10$.

Absolute Value Function

The following piecewise function is common and so we reserve the notation $|x|$ for the absolute value function:

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

Graphing Piecewise Functions

1. Graph
$$f(x) = \begin{cases} 2x & 0 < x \\ x & x \geq 0 \end{cases}$$
2. Graph
$$g(x) = \begin{cases} -5 & x < -1 \\ x & -1 \leq x \leq 1 \\ 5 & x > 1 \end{cases}$$

Modeling

Example 6. John works for the local used car dealership. His weekly salary is \$300 plus a commission based on the number of cars that he sells. His commission is \$50 per car for the first 12 cars sold each week. For any cars over 12 sold each week, John ears a commission of \$190 per car.

Complete piecewise function that can be used to calculate John's weekly salary, S, when he sells t cars per week.

$$S(t) = \begin{cases} & \text{if} \quad 0 \leq t \leq 12 \\ & \text{if} \quad t > 12 \end{cases}$$