1 Frog Hopping Problems

1. How many ways can a frog hop up a eight-step staircase if the frog can hop either one or four steps on each hop?
   Solution: We have the following possible ways: (1,1,1,1,4), (1,1,1,4,1), (1,1,4,1,1), (1,4,1,1,1), (4,1,1,1,1), (4,4), (1,1,1,1,1,1,1,1).

2. How many ways can a frog hop up a eight-step staircase if the frog can hop either one, two , three or four steps on each hop?
   Solution: There are 108 ways. To solve this problem, simplify the problem by using 1 step, then 2, then 3, then 4, then 5. At this point, you might notice that the number of combinations for 5 steps is the sum of the number of combinations for the 1,2,3,4 steps. As described in class for the original frog problem, we find that this works in general and so we can compute the number of ways for 8 steps by this process.

2 Cucumber Type Problems

1. You have placed in storage 150 pounds of cucumbers which are 90% water by weight. After a week or so, some evaporation has occurred whereby the cucumbers are now only 89% water by weight. What does your batch of cucumbers now weigh? Explain why removing such a small amount of water changed the weight so much.
   Solution:Let W_i mean the initial weight (before evaporation), W_f mean the final weight (after evaporation), and S mean the solid weight. Note that since the cucumbers are 90% water by weight, they are 10% solid by weight. Therefore $S = .1W_i = 15lbs$. After evaporation, the cucumbers are 89% water by weight and therefore are 11% solid by weight. Note that the amount of solid is unchanged and so we must similarly have $S = .11W_f$. It follows that $W_f = \frac{S}{.11} = 136.3636\ldots lbs$. The reason why removing such a small amount of water changes the weight so much is that 90% of weight of a large amount of weight (150lbs) means that there is a lot of water here. So removing just a percent is still removing quite a bit of water.

2. You have placed in storage 10 pounds of cucumbers which are 90% water by weight. After a week or so, some evaporation has occurred whereby the cucumbers are now only 89% water by weight. What does your batch of cucumbers now weigh? Explain why this situation is different from the previous problem.
   Solution: The process is exactly the same as before. You should get that the final weight is about 9.09lbs. The way this situation is different from the previous problem is that even though the percentages are the same, something weighing 10lbs and being 90% water means there is not *that* much water here. So removing 1% of this small amount of water amounts not removing *that* much water.

3 Price Guessing Problems

1. When a farm burned, they found a bill: 52 turkeys, $_3.9_. The fire smudge obliterated the first and last digit of the total price of the birds. What are the two faded digits and what was the price of a single turkey?
   Solution: 52*1.23=63.96

2. When a farm burned, they found a bill: 122 turkeys, $_84.2_. The fire smudge obliterated the first and last digit of the total price of the birds. What are the two faded digits and what was the price of a single turkey?
   Solution: 122*2.33=284.26

4 Cryptarithmetic

1. Find values for A, B, C so that the following makes sense:

$$
\begin{array}{r}
A \quad A \quad B \\
+ \quad B \quad B \quad A \\
\hline
C \quad C \quad C
\end{array}
$$

Find two more solutions to this problem and convince yourself this problems has *many* solutions.
Solution: Possible solution A=1,B=2,C=3

2. Find values for A, B, C, D, E so that the following makes sense:

$$
\begin{array}{r}
A \quad A \quad A \\
B \quad B \quad B \\
C \quad C \quad C \\
+ \quad D \quad D \quad D \\
\hline
A \quad A \quad A \quad E
\end{array}
$$

Solution: Possible solution A=1,B=2,C=3,D=4,E=0

5 Age Guessing

1. In 1977, Some Guy's age equaled the product of the four digits in the year he was born. What is his birth year?
Solution: Possible solution: 1923. Age=54

2. In 1909, Marget's age equaled the product of the four digits in the year she was born. What is her birth year?
Solution: Possible solution: 1861. Age=48