1 Initial Notes

What to expect:

- Graded work comes back next week.
- I have several students who still need to take the test.
- I will start posting solutions to homework, exams, and in class examples.
- Website will be restructured.

New structure:

- Teach problems for you to learn how to do
  - Go over several examples of the same type of problem.
  - Give you homework and worksheets based on these examples.
  - Test on these types of problems.
  - Goal: Learn how to apply problem solving strategies.
- Math Appreciation
  - I present a problem I find interesting and its solution.
  - Discuss the problem solving techniques used.
  - Goal: Understand/appreciate problem solving strategies applied to larger problems.

2 Cryptarithmetic

Definition 1 (Cryptography)**.** *Basically the study of breaking codes and encrypted messages.*

Definition 2 (Cryptarithmetic)**.** *An arithmetic puzzle where letters and arithmetical symbols are used to disguise an arithmetic computation. Each letter must take a value from 0,1,2,3,4,5,6,7,8,9. Leading letters cannot be 0.*

Example 1. Find values for E,L,F,O, so that the following makes sense:

$$\begin{array}{cccc} & E & L & F \\ + & E & L & F \\ \hline F & O & O & L \end{array}$$

Solution 1. 1. Guess and check

2. Consider if F=2. Do we have any issues? Can we ever choose E so that E+E makes F into a two? Note that when adding any 2 three digit numbers, the largest we can get is 1998 and so F must be 1.
3. Rewrite arithmetical equation:
$$\begin{array}{cccc} & E & L & 1 \\ + & E & L & 1 \\ \hline 1 & O & O & L \end{array}$$
4. So what *must* L be? We see that L=1+1=2.
5. Rewrite arithmetical equation:
$$\begin{array}{cccc} & E & 2 & 1 \\ + & E & 2 & 1 \\ \hline 1 & O & O & 2 \end{array}$$

6. Now what *must* O be? We see that O=2+2=4.

7. Rewrite arithmetical equation:

$$\begin{array}{cccc} & E & 2 & 1 \\ + & E & 2 & 1 \\ \hline 1 & 4 & 4 & 2 \end{array}$$

8. What *must* E be? Well, we have E+E=14. So E must be 7.

9. Rewrite arithmetical equation:

$$\begin{array}{cccc} & 7 & 2 & 1 \\ + & 7 & 2 & 1 \\ \hline 1 & 4 & 4 & 2 \end{array}$$

10. Done!

Example 2. Find values for S,O,T so that the following makes sense:

$$\begin{array}{cccc} & S & O & \\ + & S & O & \\ \hline T & O & O & \end{array}$$

Solution 2.

1. Ask yourself, What values can O take? We see that if add any number other than 0 to itself, we get a different number. E.g., 2+2=4,3+3=6,7+7=14, yet 0+0=0. Therefore, O=0.

2. Rewrite arithmetical equation:

$$\begin{array}{cccc} & & S & 0 \\ + & & S & 0 \\ \hline & T & 0 & 0 \end{array}$$

3. What values *must* S and T be? We see that we *must* have

$$\begin{array}{ccc} & & S \\ + & & S \\ \hline & T & 0 \end{array}$$

from which we can conclude S=5 and T=1.

4. Rewrite:

$$\begin{array}{cccc} & & 5 & 0 \\ + & & 5 & 0 \\ \hline & 1 & 0 & 0 \end{array}$$

5. Done!

Example 3. Find values for U,S,A,L so that the following makes sense:

$$\begin{array}{cccc} & U & S & \\ + & A & S & \\ \hline A & L & L & \end{array}$$

Solution 3.

1. Recall the first example. We get that A=1 since the largest 2 two digit numbers can add up to is 198.

2. Rewrite:

$$\begin{array}{cccc} & & U & S \\ + & & 1 & S \\ \hline & 1 & L & L \end{array}$$

3. Here we will need to do some guessing and checking. We know we need to get a three digit number, so we guess U=9 since

$$\begin{array}{ccc} & & 9 \\ + & & 1 \\ \hline & 1 & 0 \end{array}$$

However, this gives us L=0 and so

$$\begin{array}{cccc} & & 9 & S \\ + & & 1 & S \\ \hline & 1 & 0 & 0 \end{array}$$

Then, since we never said each two letters cannot have the same value, we can see that S=0 works. So

$$\begin{array}{cccc} & & 9 & 0 \\ + & & 1 & 0 \\ \hline & 1 & 0 & 0 \end{array}$$

4. Let's look at the other solution. Let's guess S=5. Note L must equal zero. Then

$$\begin{array}{cccc} & & U & 5 \\ + & & 1 & 5 \\ \hline & 1 & 0 & 0 \end{array}$$

5. Now what must U be to make this work? We see that we get U+1+1=10, since with S=5, we had to carry a one. Thus U=8. Therefore, we see

$$\begin{array}{cccc} & & 8 & 5 \\ + & & 1 & 5 \\ \hline & 1 & 0 & 0 \end{array}$$

Example 4. Find values for C,O,A,L,S,I so the following makes sense:

$$\begin{array}{ccccc} & C & O & C & A \\ + & C & O & L & A \\ \hline O & A & S & I & S \end{array}$$

Solution 4. 1. Like usual, we find that O=1.

2. Rewrite:

$$\begin{array}{ccccc} & C & 1 & C & A \\ + & C & 1 & L & A \\ \hline 1 & A & S & I & S \end{array}$$

3. It follows that 1+1=S and so S=2.

4. Rewrite:

$$\begin{array}{ccccc} & C & 1 & C & A \\ + & C & 1 & L & A \\ \hline 1 & A & 2 & I & 2 \end{array}$$

5. Here we have to guess and check for the values of A. Suppose we guessed A=1. Then we have the following:

$$\begin{array}{ccccc} & C & 1 & C & 1 \\ + & C & 1 & L & 1 \\ \hline 1 & 1 & 2 & I & 2 \end{array}$$

This requires C+C=11, which is impossible since 11 is not even. Therefore, we must have A=6.

6. Rewrite:

$$\begin{array}{ccccc} & C & 1 & C & 6 \\ + & C & 1 & L & 6 \\ \hline 1 & 6 & 2 & I & 2 \end{array}$$

7. From this we see that C+C=16 and so C=8.

8. Rewrite:

$$\begin{array}{ccccc} & 8 & 1 & 8 & 6 \\ + & 8 & 1 & L & 6 \\ \hline 1 & 6 & 2 & I & 2 \end{array}$$

9. Note that since A=2, O=1 and the 6+6=12 forces us to carry a 1, we get 8+L+1 cannot be larger than 9. Note that if we did use L=1 and I=9, we would have:

$$\begin{array}{cccccc} & 8 & 1 & 8 & 6 \\ + & 8 & 1 & 1 & 6 \\ \hline 1 & 6 & 2 & 9 & 2 \end{array}$$

which doesn't work.
Therefore, we must have L=0 and I=9 to get:

$$\begin{array}{cccccc} & 8 & 1 & 8 & 6 \\ + & 8 & 1 & 0 & 6 \\ \hline 1 & 6 & 2 & 9 & 2 \end{array}$$

3 Guess the Age

In this section we will focus on a specific type of problem where the goal is to find the age of someone. Some of these problems might be able to be solved with algebra, but we will focus on the guess and check method with some reasoning.

Example 5. In 1996 my grandfather's age equaled the product of the four digits in the year he was born and the same was true for his daughter. Suppose the grandfather was born in the 20th century. Find the two birth years.

Solution 5. Let's write G for grandfather. Let's write D for daughter.

1. Let's try to grasp the problem. We know the G was born in the 20th century, so his birthday looks like 19xx.
2. Since birthday looks like 19xx, G's age must be a multiple of 9.
3. Let's guess and check some reasonable multiples of 9. E.g., 90, 81, 72, 63, and maybe 54 seem reasonable, but 18 does not.
4. Guess 90. Then G was born on the year 1996-90=1906. But this means his age is 1*9*0*6=0.
5. Guess 81. Then G was born on the year 1996-81=1915. Then age=1*9*1*5=45.
6. Guess 72. Then G was born on the year 1996-72=1924. Then age=1*9*2*4=72.
7. Now we need to find the age of the daughter. Again, her age must be a multiple of 9.
8. Let's guess and check some reasonable multiples of 9. Let's say, e.g., 27, 36, 45, 56.
9. Guess 27. Then D was born on 1996-27=1969. Clearly doesn't work.
10. Guess 36. Then D was born on 1996-36=1960. Doesn't work.
11. Guess 45. Then D was born on 1996-45=1951. Then age=1*9*5*1=45.
12. Done!

Example 6. In 1765, Adam Smith's age equaled the product of the four digits in the year he was born. What is his birth year? He was born in the 18th century.

Solution 6. Let A mean Adam Smith.

1. Like before, we know that A's age must be a multiple of 7.
2. This time we don't really know what a good guess is. Let's just start with 21.
3. Guess 21. Then A was born on 1765-21=1744. Then $age = 1 * 7 * 4 * 4 > 21$.

4. Guess 28. Then A was born on 1765-28=1737. Then $age = 1 * 7 * 3 * 7 > 28$.

5. Guess 35. Then A was born on 1765-35=1730. Then age=1*7*3*0=0.

6. Guess 42. Then A was born on 1765-42=1723. Then age=1*7*2*3=42.

7. Done!

Example 7. In 1715, Isaac Newton's age equaled the product of the four digits in the year he was was born. What is his birth year? He was born in the 17th century.

Solution 7. Let I mean Issac Newton.

1. Again, I's age must be a multiple of 6 subtracted by 2.

2. Let's guess some multiples of 6: 48, 54, 60, 66, 72

3. Guess 48. Then I was born on 1715-48=1667. Then age=1*6*6*7> 48.

4. Guess 54. Then I was born on 1715-54=1661. Then age=1*6*6*1=36.

5. Guess 60. Then I was born on 1715-60=1655. Then age=1*6*5*5=150.

6. Guess 66. Then I was born on 1715-66=1649. Then age=1*6*4*9> 66.

7. Guess 72. Then I was born on 1715-72=1643. Then age=1*6*4*3=72.

8. Done!

4 Stir It Up Lecture

For the next couple of lectures we will be covering the strategy in the book called *Stir it up*. The book describes it as "Getting your hands dirty", "to put your spoon into the problem stew and churn." There are several substrategies associated with *Stir it up* that we will see. The first substrategy will shall consider is:

Guess and Check
This is a classic way to solve problems, to guess and check.
You try different answers until you come up with the right one.
For some problems, this is the only way to solve them.

Experiment
By this, I mean run actual experiments, gather data, and then analyze data to gain insights.

Shed the Cocoon
We wrap ourselves in a cocoon of inhibitions, misconceptions, prejudices.
Common mistake is to put unnecessary constraints on our efforts to solve problems: fear of looking stupid, failing, etc.
Once you're able to shed such a cocoon, problem solving (and life in general) becomes much easier and enjoyable.

Work Backwards
Start with what you think is the last step of a solution and ask yourself, "What do I need to know or do mathematically to arrive at this last step?"
If you can answer this question, continue the process and work back to the beginning.
This is especially useful for problems with many stages.

This lecture we have primarily focused on the first substrategy of *Guess and Check*. We will see in another lecture the use of *Experiment*. Concerning the third strategy, I hope you take it as general advice and keep it with you. Lastly, if we have time, we will see the last strategy mentioned in action.