Exam 2

Do **ONLY FOUR** out of **FIVE** of the following problems. I will only grade the first four problems in your bluebook, so cross out or write "Omit" for anything you do not want graded. You may use a calculator, but cell phones are not allowed. Good luck!

Problem 1. *(25 points) For these problems, letters may take only values from 0,1,2,3,4,5,6,7,8, and 9, unless the letter is a leading letter, in which it cannot be 0. Different letters may take on the same value. Justify.*

(a) (15 points) Find one solution to the following Cryptarithmetic problem:

$$\begin{array}{cccccc} & C & O & C & A \\ + & C & O & L & A \\ \hline O & A & S & I & S \end{array}$$

***Solution**: The solution to this is in the Oct6 notes.*

(b) (10 points) Explain why the following Cryptarithmetic is unsolvable:

$$\begin{array}{cccccc} & & F & O & U & R \\ & + & F & O & U & R \\ \hline T & W & E & L & V & E \end{array}$$

***Solution**: The largest possible four digit number is 9999 and so the largest possible number FOUR+FOUR can be is 9999+9999=19998. Note that 19998 has five digits, yet TWELVE has 6 digits. Therefore FOUR+FOUR can never add up to TWELVE.*

Problem 2. *(25 points)*

(a) (10 points) How many ways can a frog hop up a four-step staircase if the frog can hop either one, two, or three steps per hop?
***Solution**: We have as the possible combinations: (1,1,1,1), (1,1,2), (1,2,1), (2,1,1), (2,2), (1,3), and (3,1). Therefore there are 7 possible ways. It wasn't necessary to simply this part by considering 1, 2, and 3 steps and then seeing a pattern, though if it helped, simplification was allowed.*

(b) (10 points) How about if the frog can hope 1 or 3 steps per hop?
***Solution**: For this one you just needed to enumerate the possibilities: we have (1,1,1,1), (1,3) and (3,1). So there are 3 possible ways.*

(c) (5 points) Explain why the answer to (b) is less than the answer to (a).
***Solution**: The reasoning here was that by removing the ability to do 2 steps per hop, we have eliminated all combinations with a "2" in it (e.g., (2,1,1)). However, we have kept the remaining combinations with a "1" in it (e.g., (1,1,1,1)), and all the combinations with a "3" in it (e.g., (1,3)). Therefore, by removing the ability to hop 2 steps per hop, we have removed 4 possible combinations.*

Problem 3. *(25 points) You have placed in storage 200 pounds of cucumbers which are 99% water by weight. After a week or so, some evaporation has occurred whereby the cucumbers are now only 98% water by weight.*

(a) (15 points) What does your batch of cucumbers now weigh?
***Solution**: Let's start out by creating some variables. Let W_i mean the initial weight (i.e., $W_i = 200lbs$, the weight before evaporation). Let W_f mean the final weight (i.e., the weight after evaporation). Let W_s mean the solid weight. Then, since before evaporation the batch is 99% water by weight, we have that the batch is 1% water by weight. It follows $W_s = 0.1 \times W_i = 0.1 \times 200 = 20lbs$; i.e., the solid portion of the batch weighs 20lbs. Keep in mind that W_s, the solid weight, remains constant throughout evaporation. Next, after evaporation, we have that the batch is 98% water by weight and is so 2% solid by weight. It follows that $0.2W_f = W_s$. We can solve for W_f to get $W_f = W_s/0.2 = 20/.2 = 100lbs$; i.e., the final weight is 100lbs.*

(b) *(5 points) What is the main assumption about the contents of the cucumber that is needed for this problem?*
***Solution**: The main assumption about the contents of the cucumber is that the solid weight of the cucumber remained constant throughout the evaporation.*

(c) *(5 points) Why is there such a large decrease in weight despite the small decrease in percentage of water by weight?*
***Solution**: The reasoning for this is that even though we have only removed a single percent of water by weight, 99% of 200lbs is quite a lot of weight. In other words, removing 1% of a large quantity is still removing quite a bit.*

Problem 4. *(25 points) In 1360, Rabid Goat's age equaled the product of the four digits in the year he was born. He was born in the 14th century. He is at least 18 years old. What is his birth year? Justify and explain as much as possible (e.g., "This guess doesn't work because BLAH").*
***Solution**: To solve this, you really just needed to guess and check. Note that being born in the 14th century allows us to conclude Goat's birth year looks like 13xx. So, we know that his birth year must be a multiple of 3. Therefore, we just guess some multiples of 3 greater than 18 and see if they work. The answer is that Goat's age is 27 and his birth year is 1333. To see this, we guess his age is 27. Then his birth year is 27 years before 1360: birth year = 1360-27=1333. Then knowing his age equals the product of the four digits in the year he was born, we get: age=1*3*3*3=27. So, him being 27 works and his birth year being 1333 works!*

Problem 5. *(25 points) Explain as much as possible for the following problem:*
Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits. Suppose that our rabbits never die and that the females always produces one new pair (one male, one female) every month from the second month on. How many pairs will there be in one year?
Hint:
1. At the end of the 1st month, they mate, but there is still one only 1 pair.
2. At the end of the 2nd month the female produces a new pair, so now there are 2 pairs.
3. At the end of the 3rd month, the original female produces a second pair, making 3 pairs.

Solution: A sketch of this solution is in the Oct15 notes.

1 Extra credit

Problem 6. *(5 points) Convert 17km to miles.*
***Solution**: Was hoping to see an application of the Fibonacci sequence!*

Problem 7. *(5 points) Solve the following Cryptarithmetic problem*

$$\begin{array}{cccccc} & B & A & S & E \\ + & B & A & L & L \\ \hline G & A & M & E & S \end{array}$$

***Solution**: Note that $B = 5$, $G = 1$, and zero for everything else works:* $5000 + 5000 = 10000$.