The following problems may help you prepare for the exam. For the class period, I will not give solutions or answers. The purpose of this exercise is to practice writing clear explanations, not coming up with precisely the correct answer. I will only tell you if you are at least mostly correct, possibly point you in the correct direction, or possibly tell you if you are completely wrong. Remember, part of getting better at something is seeing where you go wrong and correcting it! It's also more important to practice the problem solving techniques here. Remember, you should write your solutions so that if someone were to read your explanation, they would not need to ask you any clarifying questions at all.

Problem 1. Suppose you have a heap of beans of 16 beans and each player may remove 1 or 2 beans. What is the winning strategy? I want a detailed explanation on how to win and why this strategy works.

Problem 2. (Hard problem) Suppose you have a heap with n beans and each player may take 1 or 2 Describe the values of n where the strategy in the previous problem will work.

Problem 3. Two straight fences meet at an angle of 30^o and are each of length 10ft. A goat is tried by a 12 foot rope to the corner so it is "outside" the region enclosed by the fence.

1. Draw the circular regions the goat can roam.
2. Of the largest circular region, how much of this circle can the goat not roam? (I.e., how much does a 30^o slice cut out of the circle.)
3. What is the roaming area for the goat?
4. What if the angle was 40^o? How about 50^o?

Problem 4. Suppose there is a 15 foot long fence and there is a fence post three feet from one end, on the fence. Suppose there is a goat tied to this post by a 6 foot rope. Find the roaming area.

Problem 5. (Hard problem) Suppose we have a circular silo of radius 10ft and there is a goat tied to the silo by a rope of length 10ft. Draw what you would think the roaming region *would* be and explain why this problem is too difficult.

Problem 6. How many ways can you change 17 cents? How many ways can you change 26 cents? How many ways can you change 25 cents?

Problem 7. Describe the symmetries of a rectangle that is not a square.

Problem 8. (Hard problem) Describe the symmetries of a triangle on a sphere; i.e., the triangle cannot be removed from the sphere. Remember a symmetry can be a rotation, reflection, or a translation.

Problem 9. *The Green's Party.*
The Greens were having a party and were unsure how many guests they had invited. The night of the party, the first time the doorbell rang, 1 guest entered the house. On the second ring, 3 guests entered. On the third ring, 5 guests entered. As it turned out, on each successive ring the entering group was 2 guests larger than the previous group. How many guests entered on the 15th ring and how many guests in total were in the house after the 15th ring?

Do the same problem with the condition
"As it turned out, on each successive ring the entering group was 2 guests larger than the previous group."
is replaced with
"As it turned out, on each successive ring the entering group was 1 guests larger than the previous group."
and
"As it turned out, on each successive ring the entering group was 3 guests larger than the previous group."

Problem 10. In a train station waiting room you find yourself waiting for the train along with 15 other travelers. Each person has a purse with a lot of coins. Everyone decides it would be a good idea to greet each other and give each other person 3 coins upon greeting. How many coins in total will have been given? For example, if there were two people in the room, person A would give person B three coins, and person B would give person A three coins, giving us a total of six coins given.